NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

MSC INTERNAL NOTE NO. 66-FM-79

August 12, 1966

A PARAMETRIC STUDY OF CENTRAL ANGLE OF TRAVEL AND TIME FOR REENTRY FROM NEAR-EARTH ORBITS

By William R. Pruett
Flight Analysis Branch

MISSION PLANNING AND ANALYSIS DIVISION
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Mission Planning and Analysis Division

FIGURES

A PARAMETRIC STUDY OF CENTRAL ANGLE OF TRAVEL AND TIME FOR REENTRY FROM NEAR-EARTH ORBITS

By William R. Pruett

SUMMARY AND INTRODUCTION

This paper is a continuation of reference 1, "General Parametric Study for Near-Earth Orbits," by Frank J. Suler. Contained in this report are the orbit-referenced central angle of travel from retrofire to 400 000 ft and the time from retrofire to 400 000 ft as functions of true anomaly of retrofire. The same ranges of retrograde pitch angles, retrograde ΔV's, and elliptical orbits are used in this report as were used in reference 1. Since reference 1 presents reentry velocity and flight-path angle at 400 000 ft, this document and reference 1 should be used together to obtain a more valuable picture of reentry conditions.

For additional information concerning near-earth orbits, see reference 2.

MATHEMATICAL MODEL

Keplerian equations, a spherical rotating earth, and instantaneous velocity changes were used in this study. The solutions were obtained from the general elliptical orbit and reentry program, E042. Beta angles are measured positive clockwise from the local horizontal, For a geometric representation of the orbit parameters, see paragraph 2.2 of reference 1.

DISCUSSION OF RESULTS

The figures presented were plotted by an SC 4020 microfilm plotter. The plot program used (ref. 3) converts binary x-y coordinates generated by an IBM 7094 computer into 4020 plot tapes.

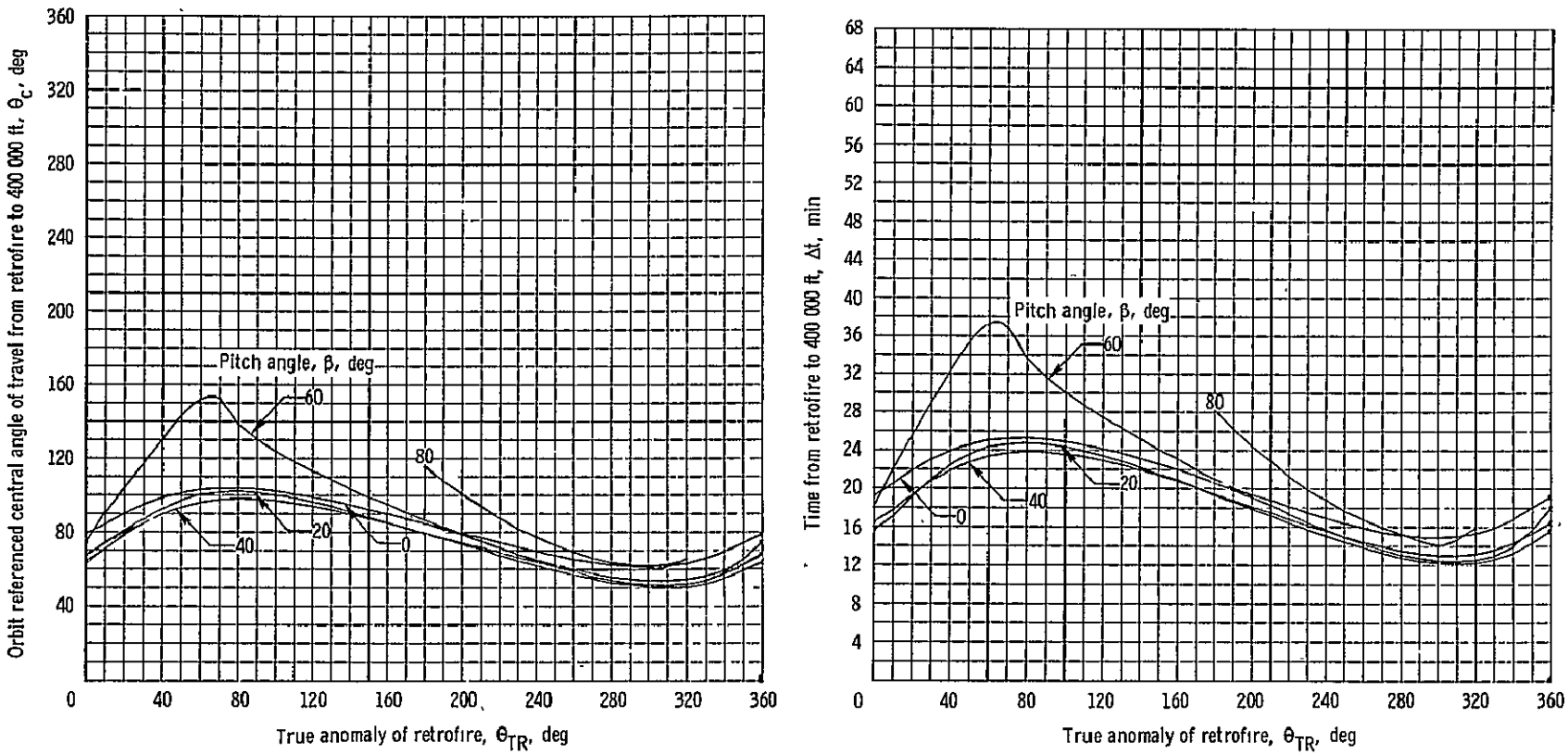
The curves plotted in the figures are not extremely smooth, for two main reasons.

First, the true anomaly increment used was ten degrees; therefore, a computed point is plotted every ten degrees. These points were simply connected by straight lines, and no attempt was made to fit a curve to them. Smoother curves could be obtained by using a smaller computing interval, but this would greatly increase the computing time. This could not be justified since the errors encountered using a 10° true anomaly increment were not significant when compared to other computation errors.

Second, errors existed because of the program's characteristics. In the program, true anomaly and time are calculated from a computed value of flight-path angle. For the very slightly elliptical orbits which were considered, flight-path angles seldom exceeded $\pm 1^{\circ}$, and calculations of true anomaly and time based on these angles were not extremely accurate. Some errors were found to be near $\pm 4^{\circ}$ in true anomaly and $\pm$ 60 seconds in time, but these errors were seldom experienced. The average error in computation, for the reason stated, was about $\pm .5^{\circ}$ in true anomaly and $\pm$ 10 seconds in time.

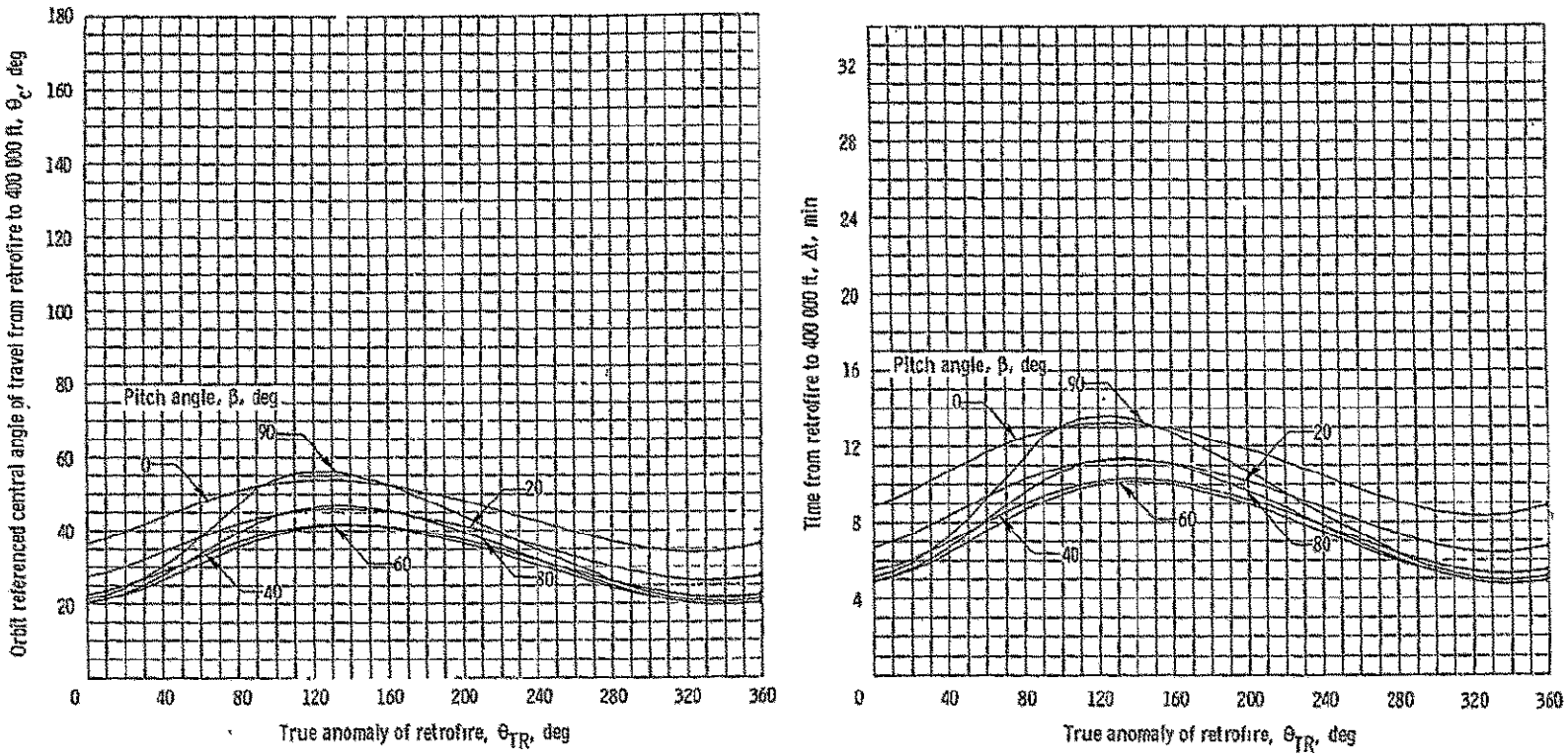
These errors were obtained by simply interpreting inconsistencies in the plots and are not meant to be highly accurate. In fact, this entire explanation of the figures is intended only to explain why the figures are not extremely smooth, not to describe how large the inaccuracies are. However, for the general mission planning for which this paper is intended, the data presented is felt to be quite satisfactory.

Each of the figures presents both time from retrofire to 400 000 ft and orbit-referenced central angle of travel from retrofire to 400 000 ft as functions of true anomaly of retrofire. Pitch angles of 0°, 20°, 40°, 60°, 80°, and 90° were used for each plot, and retrograde ΔV's of 100 fps, 300 fps, 500 fps and 700 fps were used for each figure. If one or more of these parameters are not presented in a figure, reentry was not possible under those conditions.



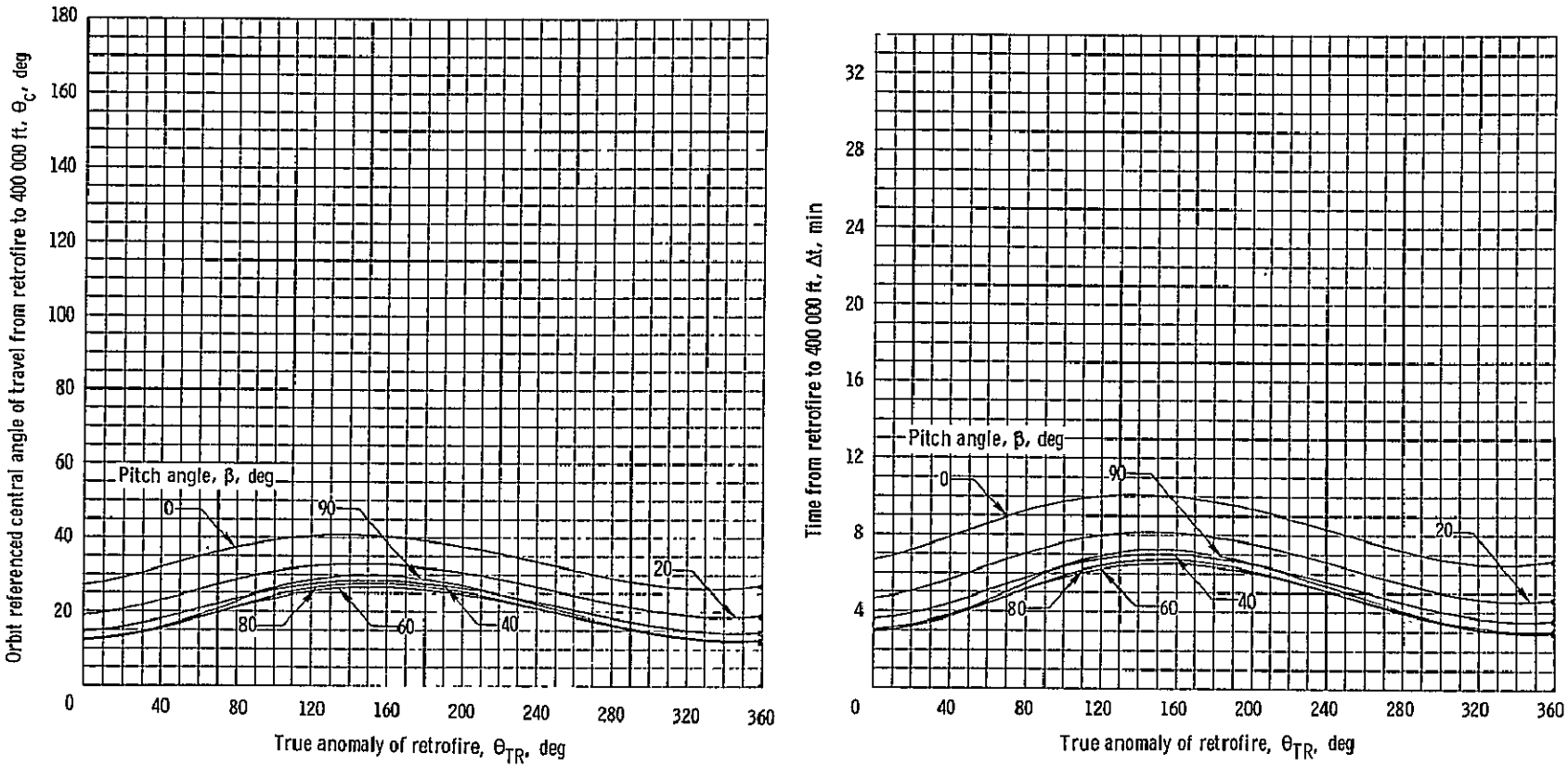
(a) Retrograde ΔV = 100 fps.

Figure 1.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 100 nautical miles and h_p = 80 nautical miles.



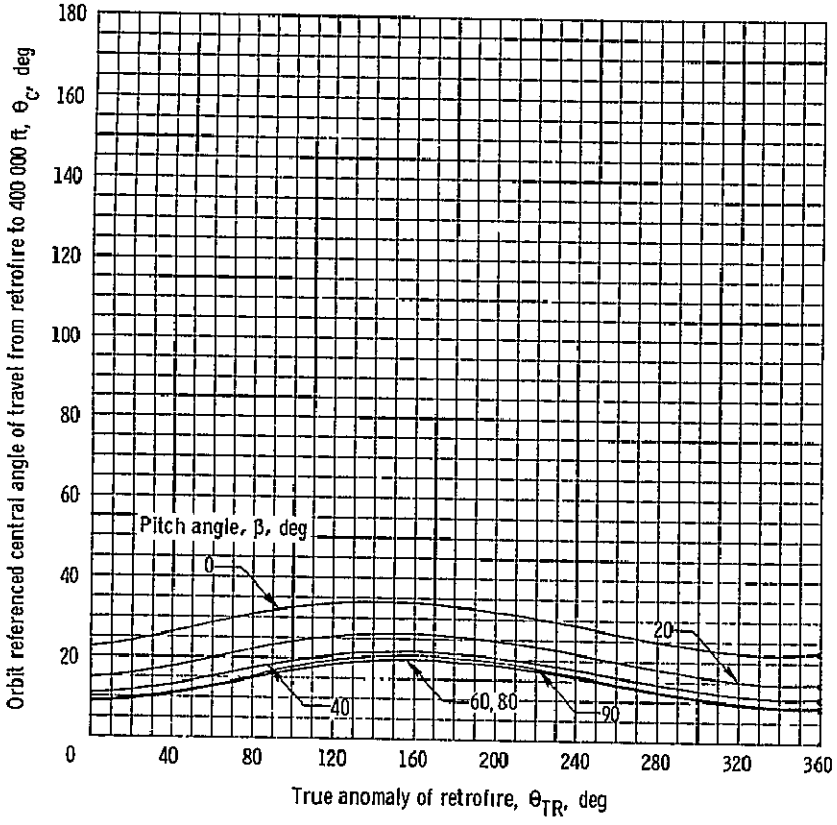
(b) Retrograde ΔV = 300 fps.

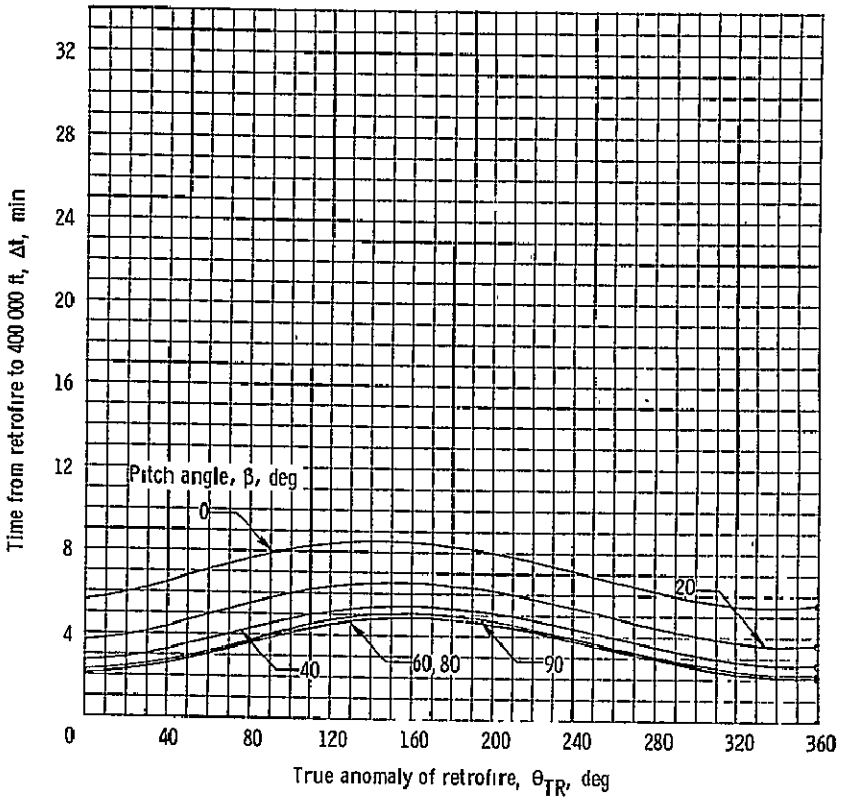
Figure 1. - Continued.



(c) Retrograde ΔV = 500 fps.

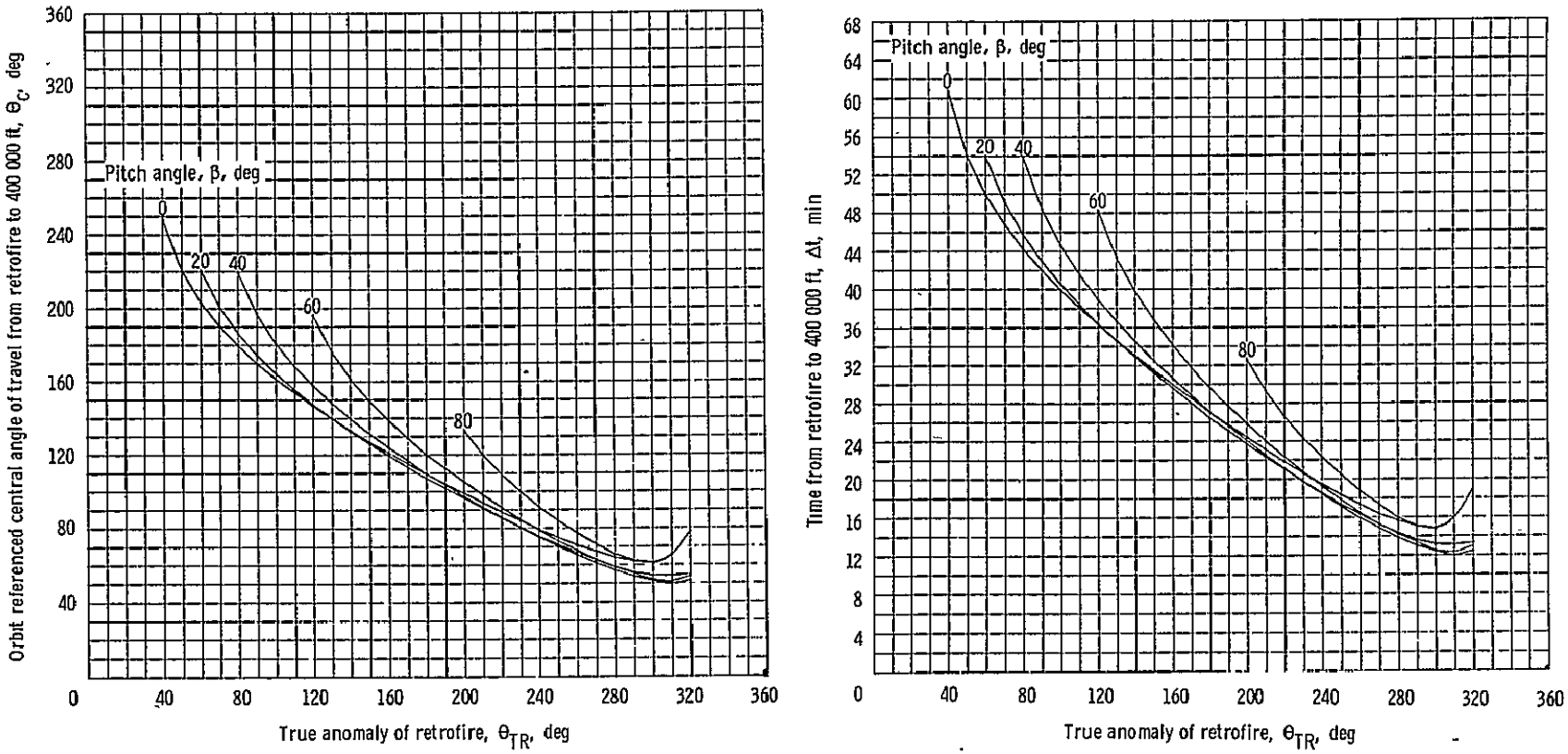
Figure 1. - Continued.





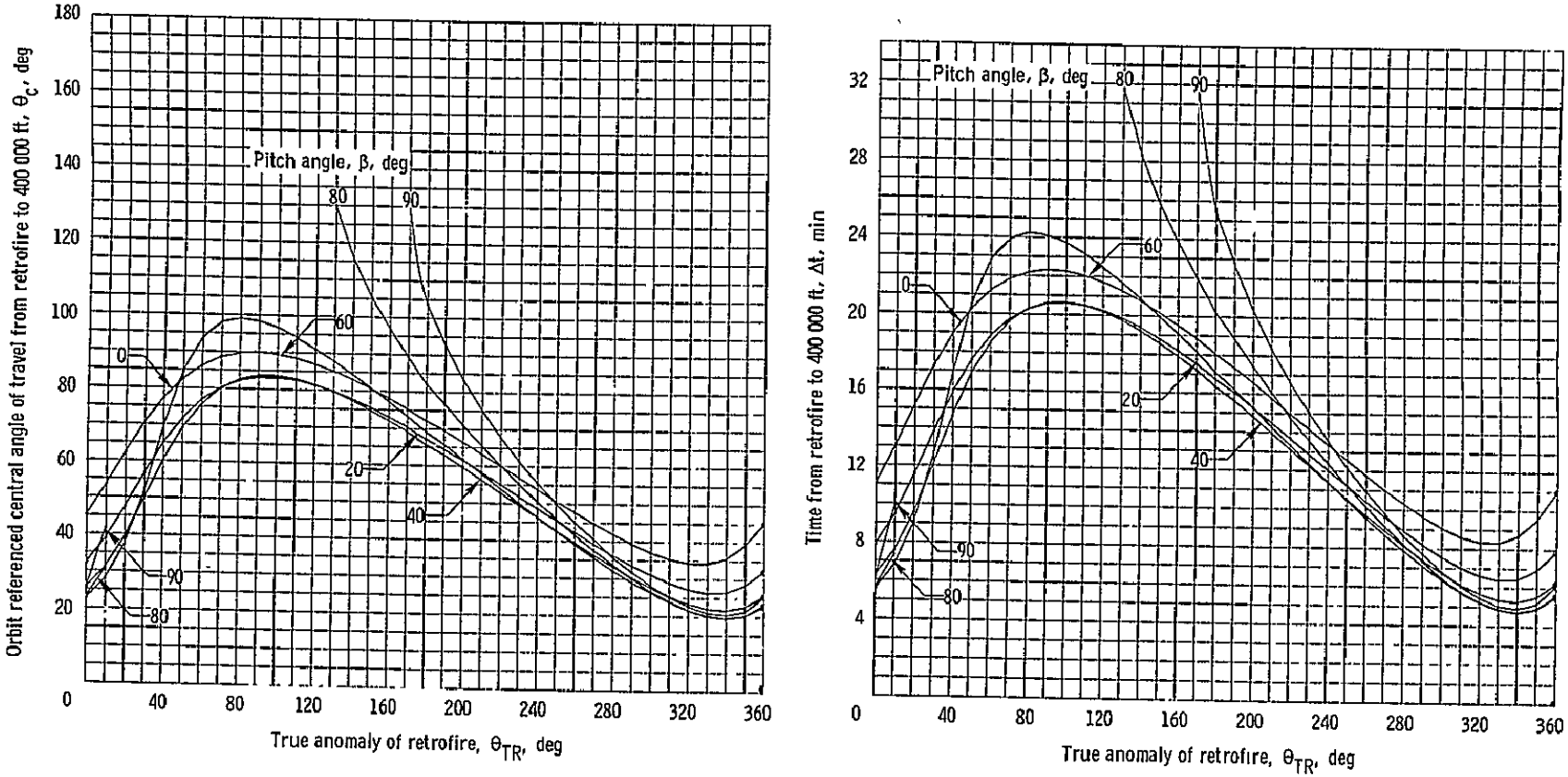
(d) Retrograde ΔV = 700 fps.

Figure 1. - Concluded.



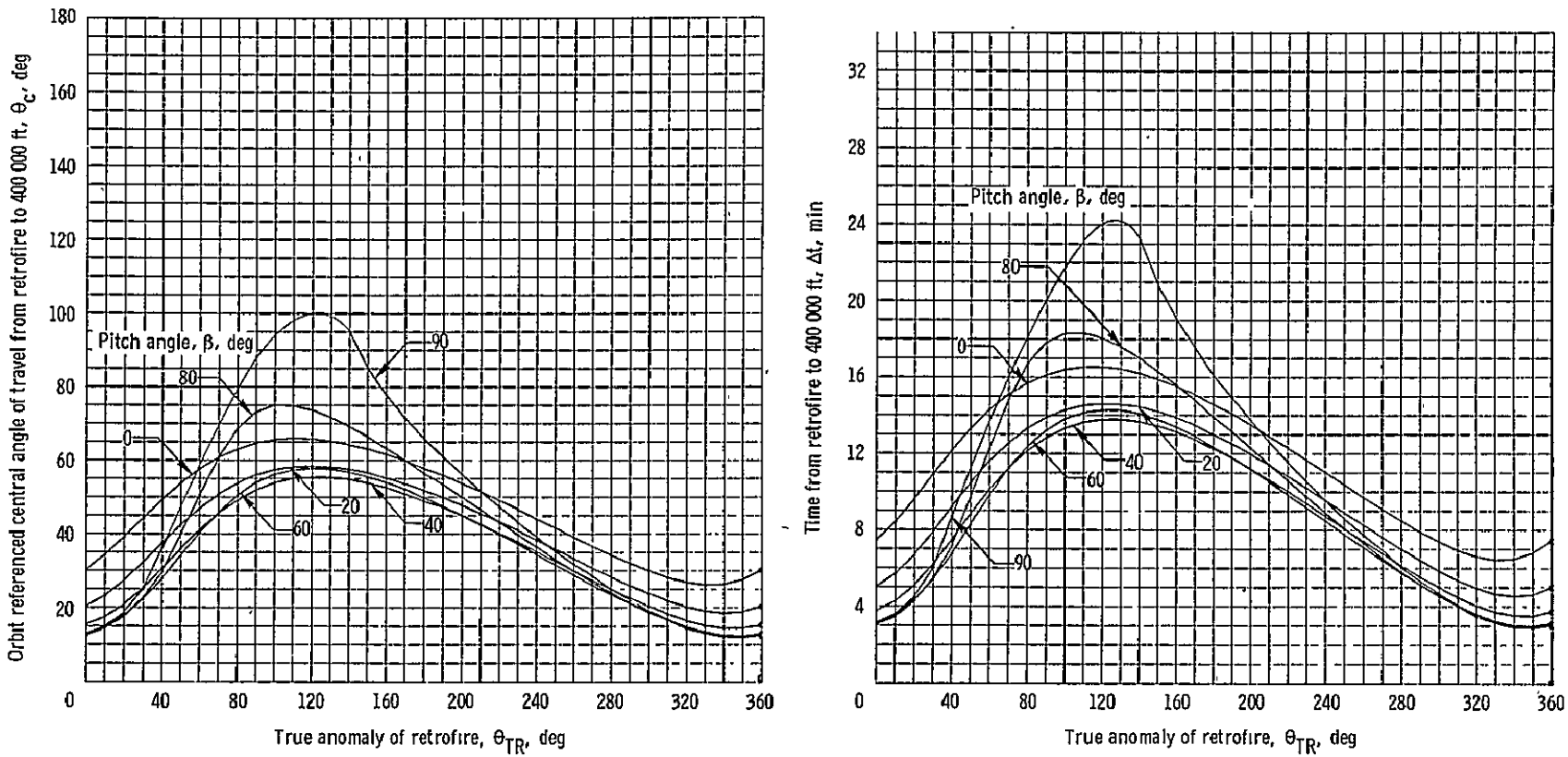
(a) Retrograde ΔV = 100 fps.

Figure 2.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 150 nautical miles and h_p = 80 nautical miles.



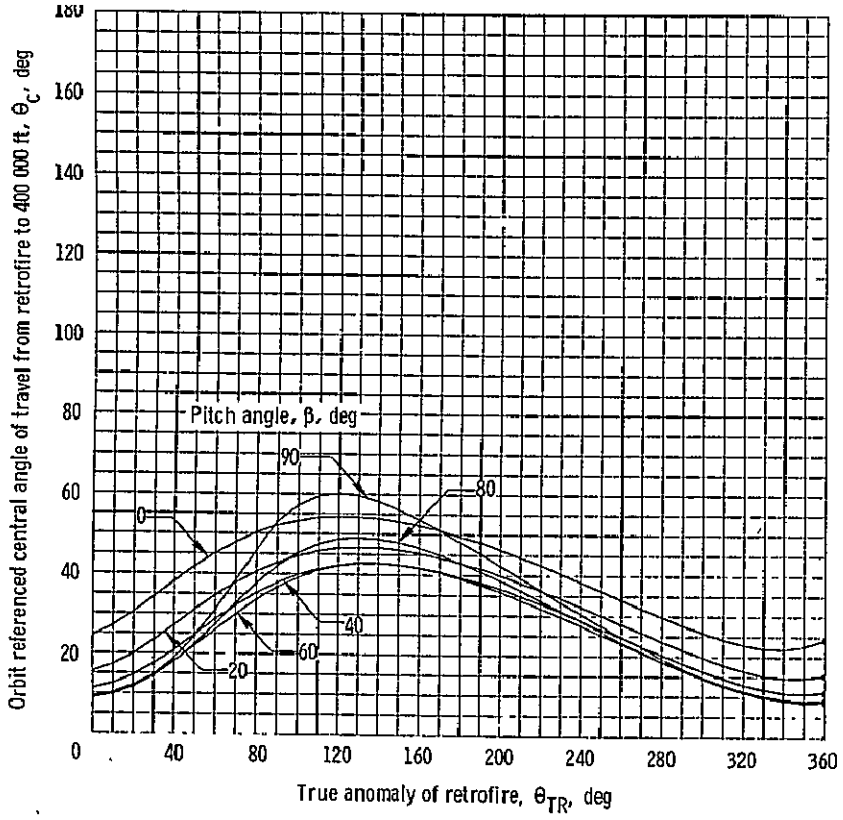
(b) Retrograde ΔV = 300 fps.

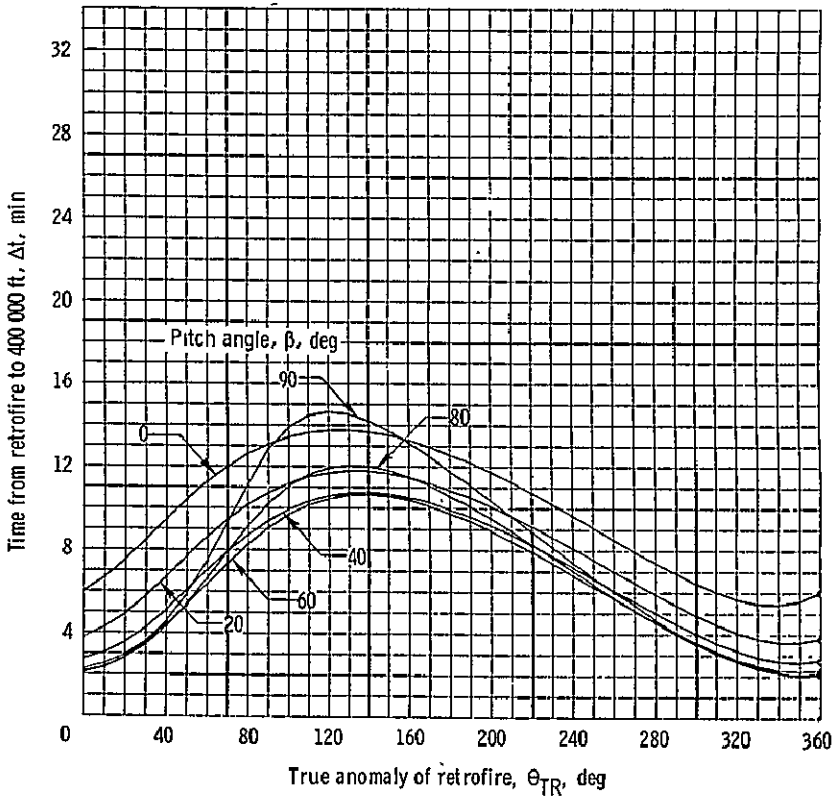
Figure 2. - Continued.



(c) Retrograde ΔV = 500 fps.

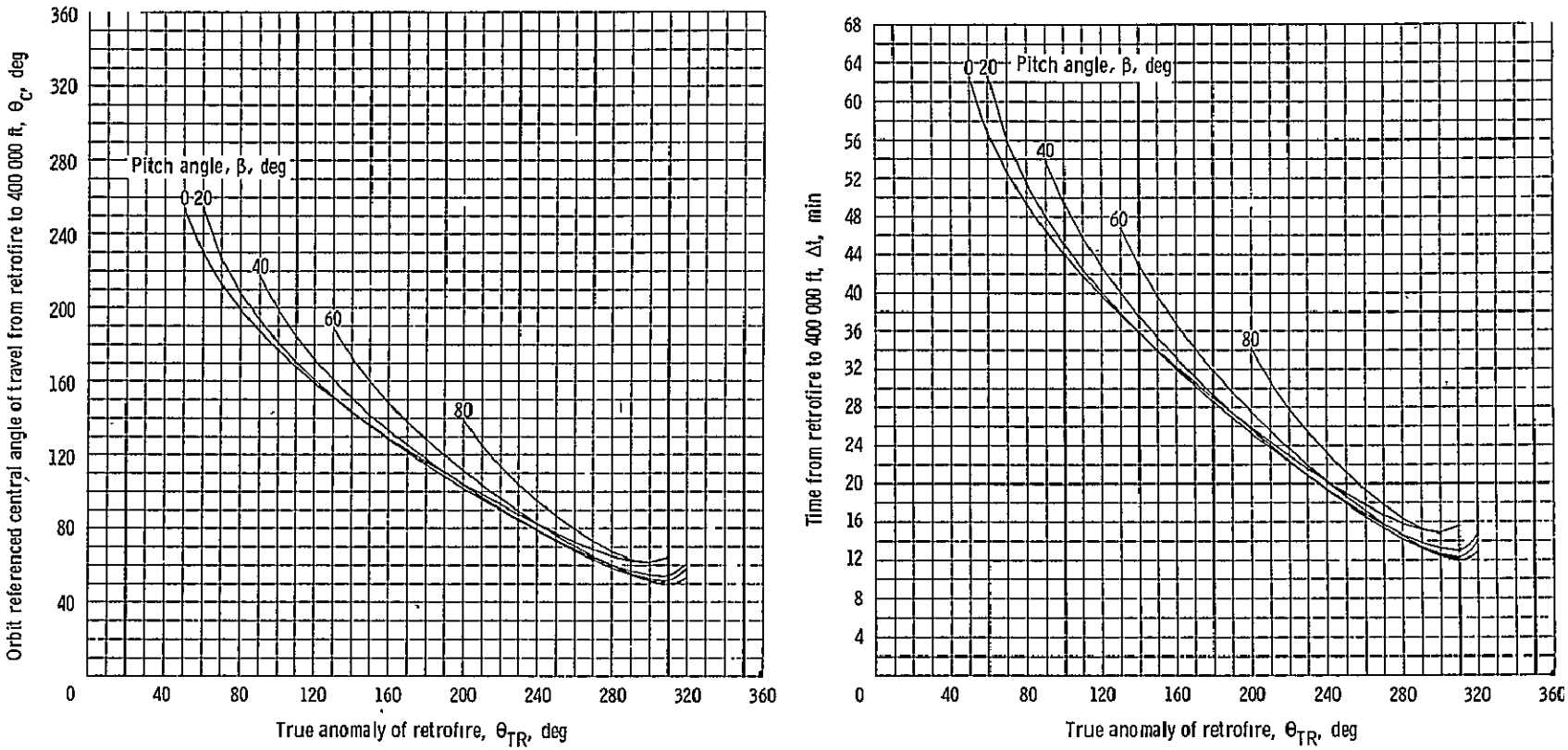
Figure 2. - Continued.





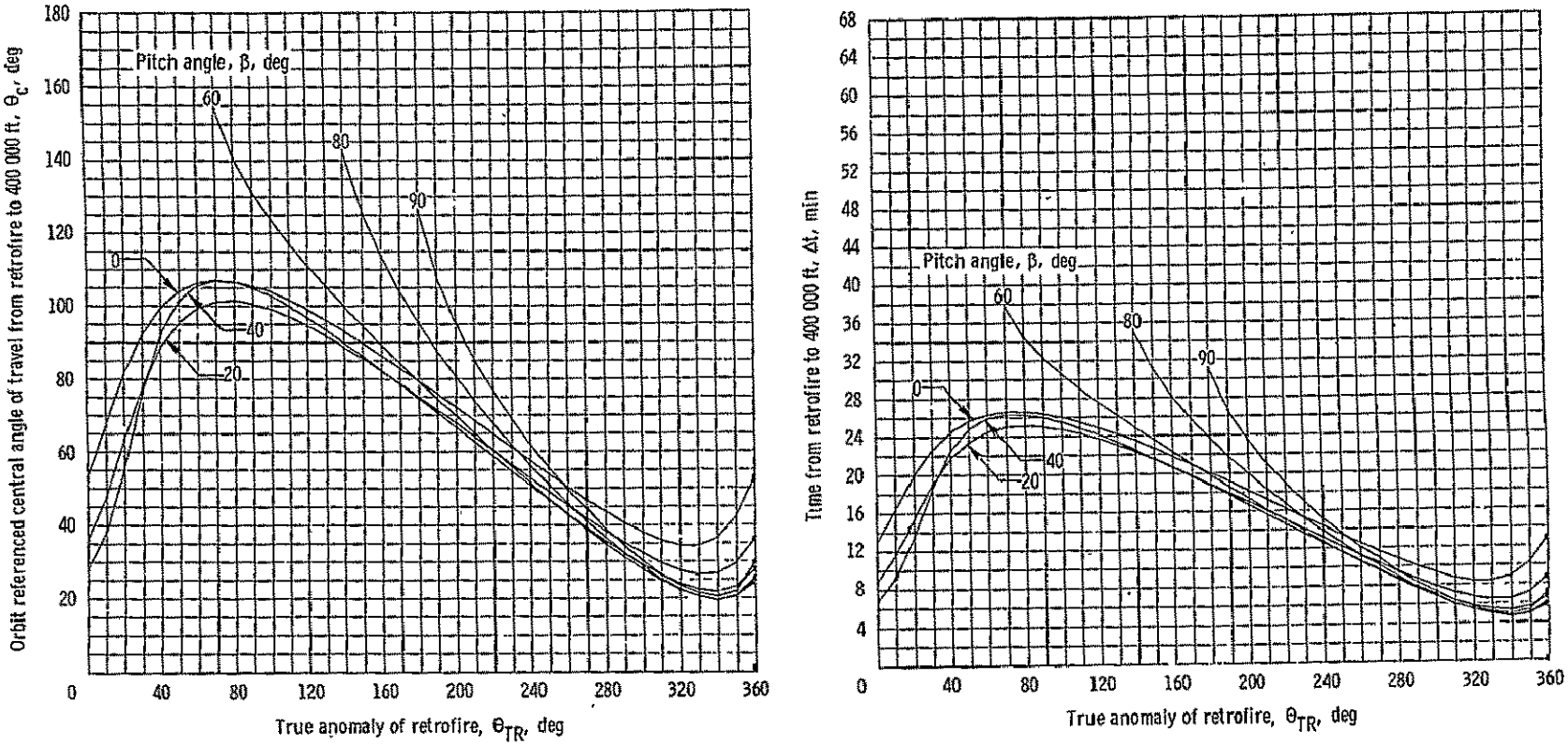
(d) Retrograde ΔV = 700 fps.

Figure 2. - Concluded.



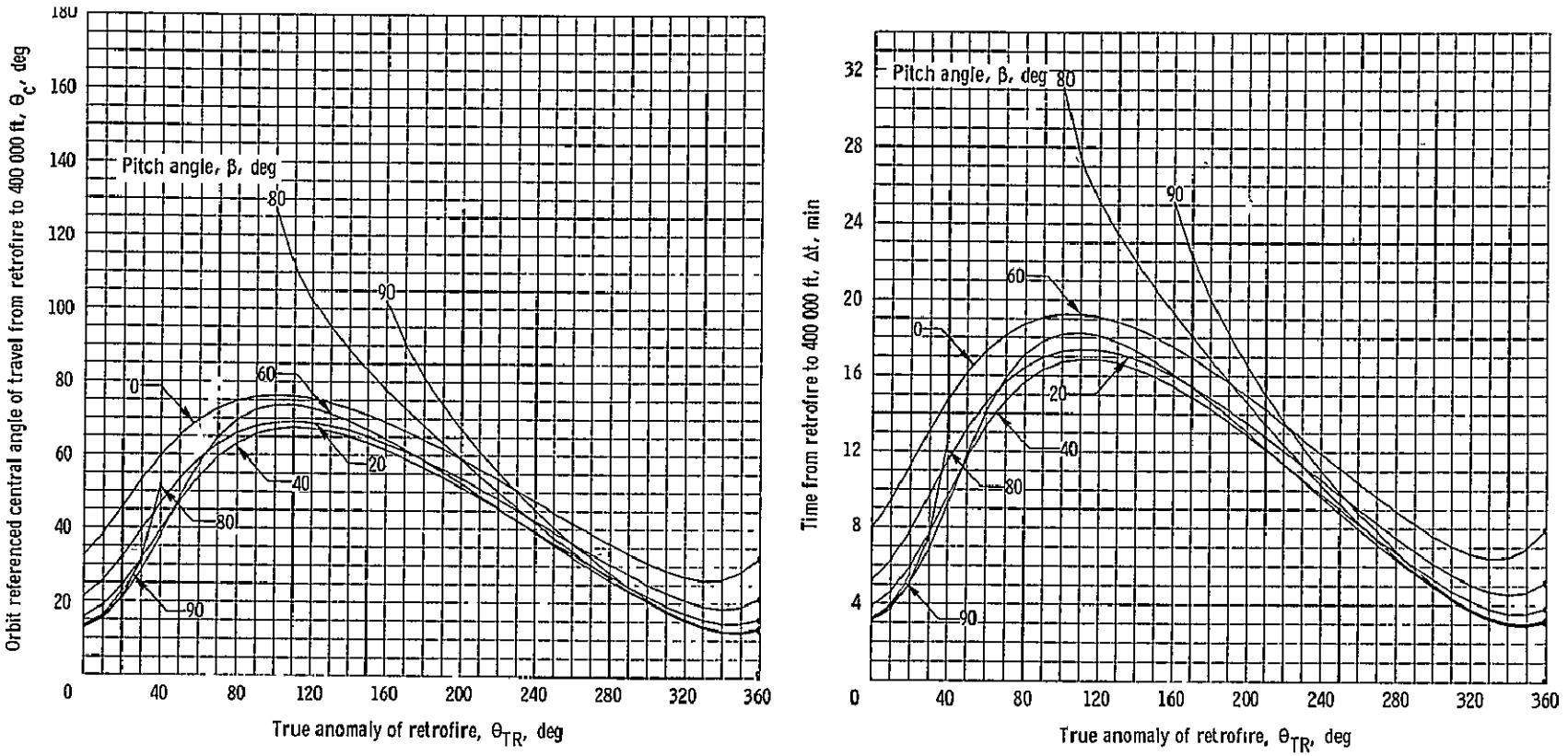
(a) Retrograde ΔV = 100 fps.

Figure 3. – Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 175 nautical miles and h_p = 80 nautical miles.



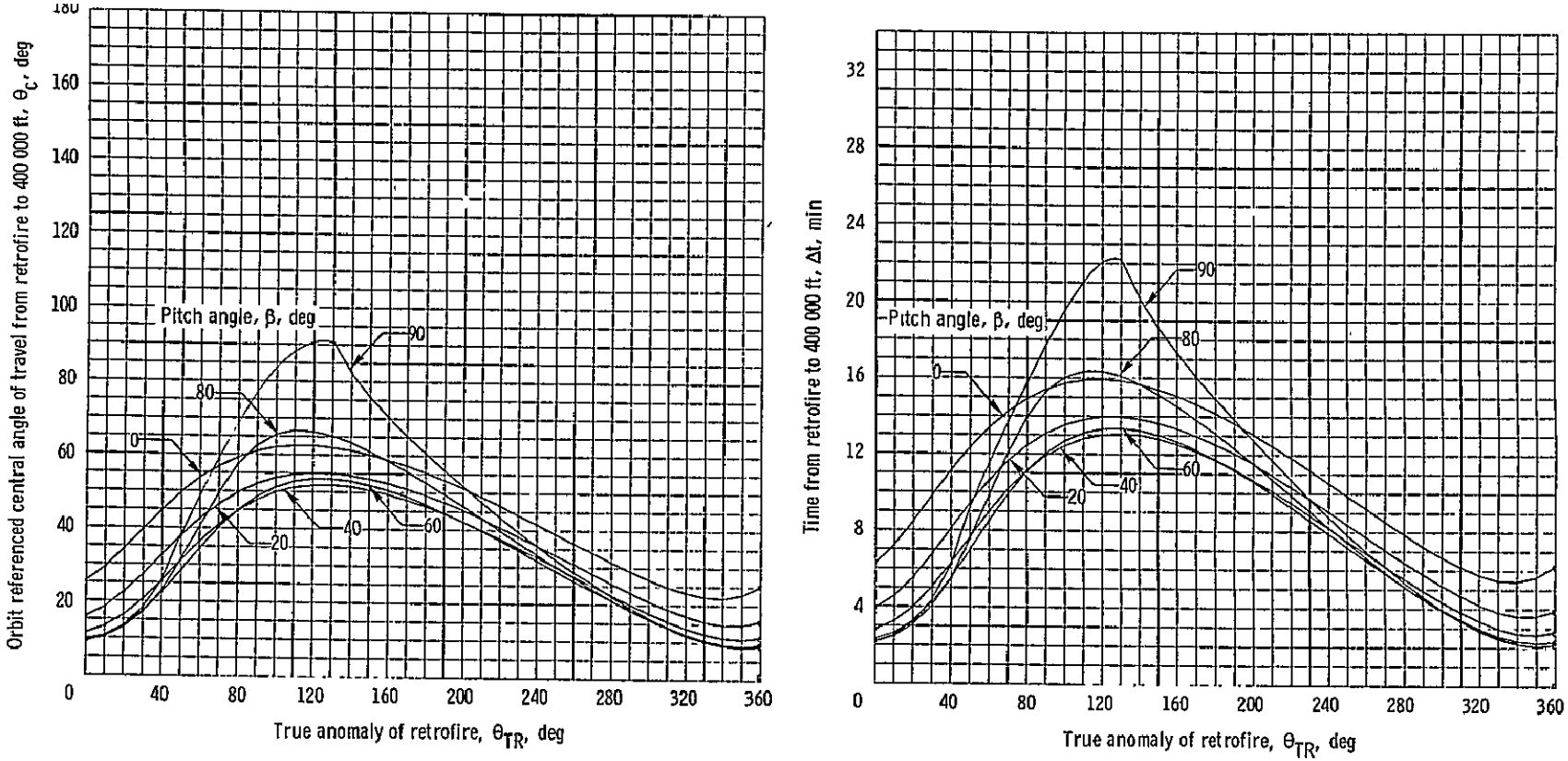
(b) Retrograde ΔV = 300 fps.

Figure 3. - Continued.



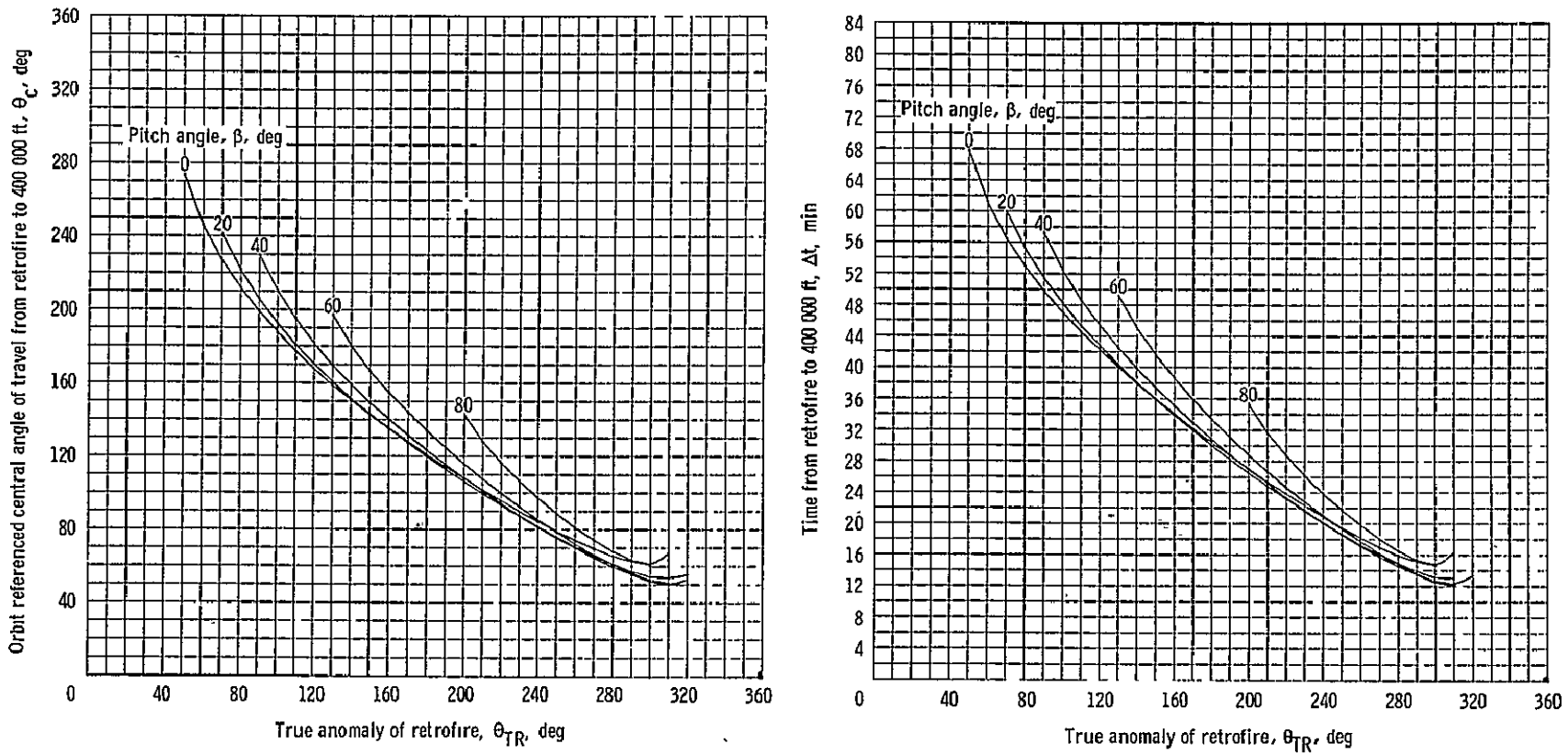
(c) Retrograde ΔV = 500 fps.

Figure 3. - Continued.



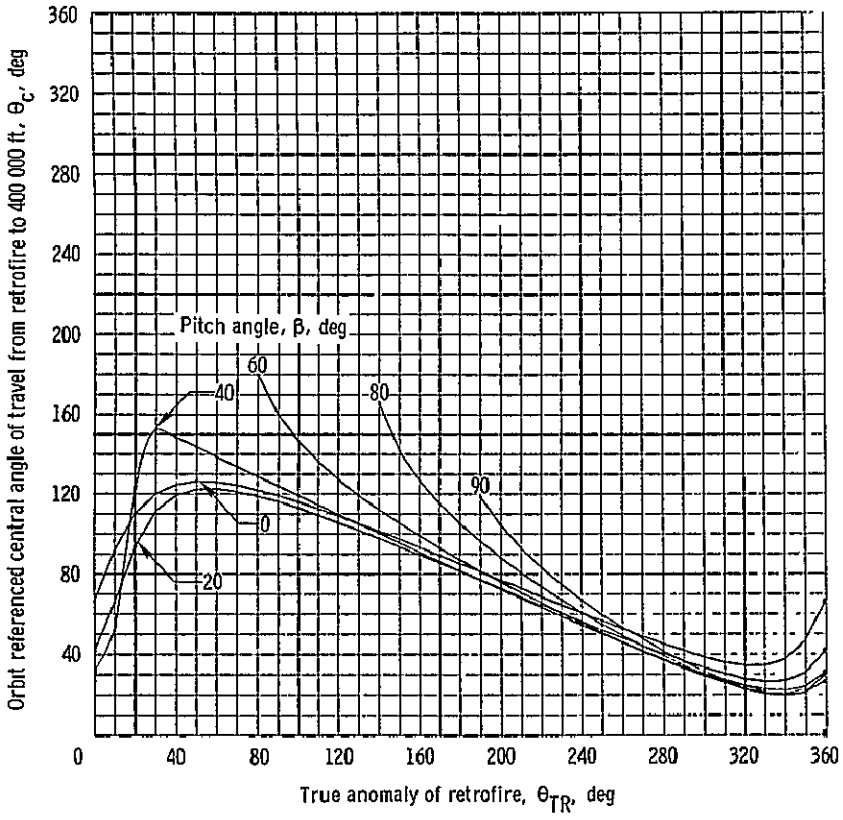
(d) Retrograde ΔV = 700 fps.

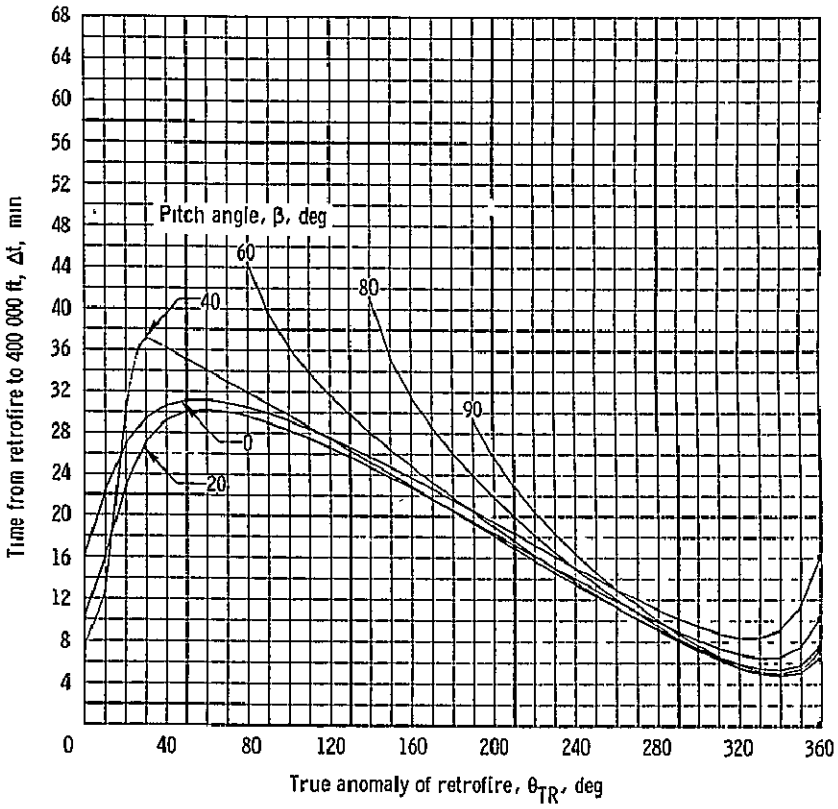
Figure 3. - Concluded.



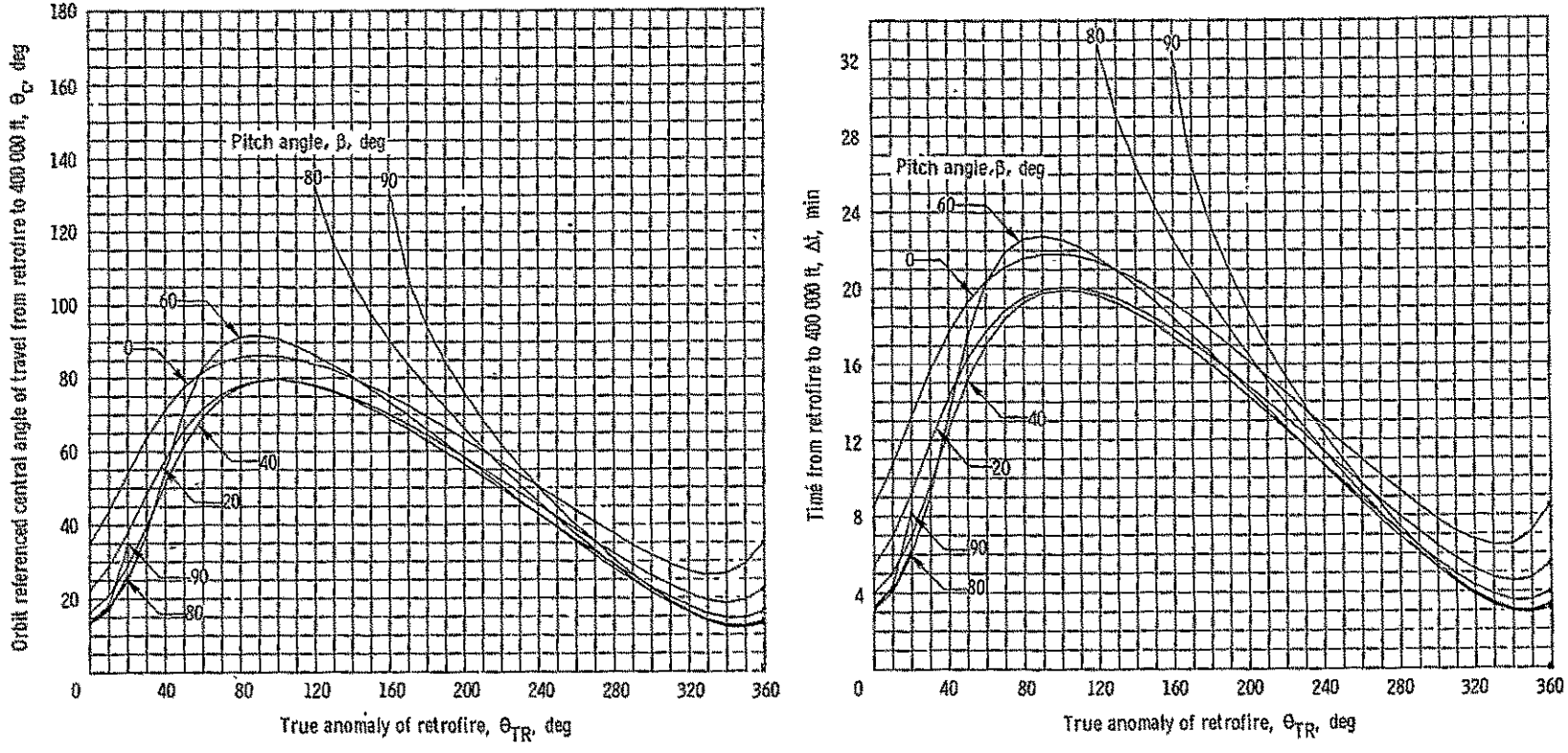
(a) Retrograde ΔV = 100 fps.

Figure 4.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 200 nautical miles and h_p = 80 nautical miles.



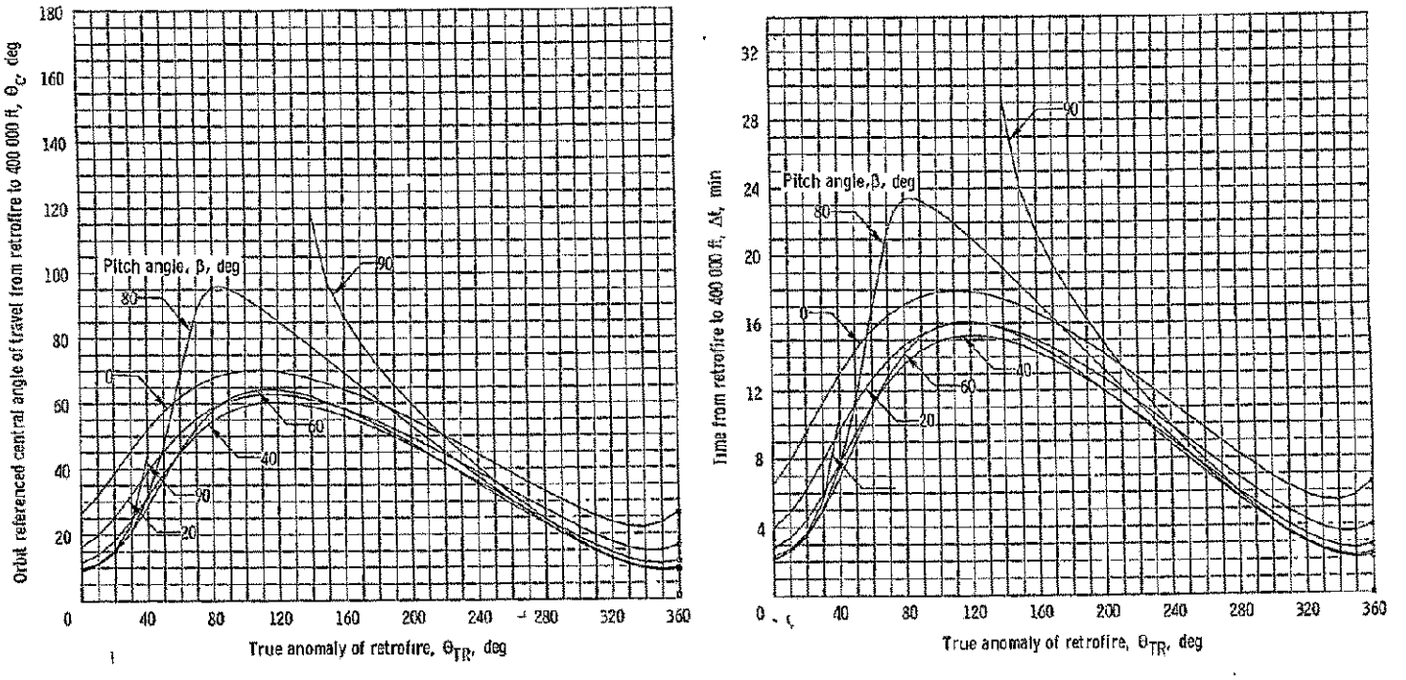


(b) Retrograde ΔV = 300 fps.



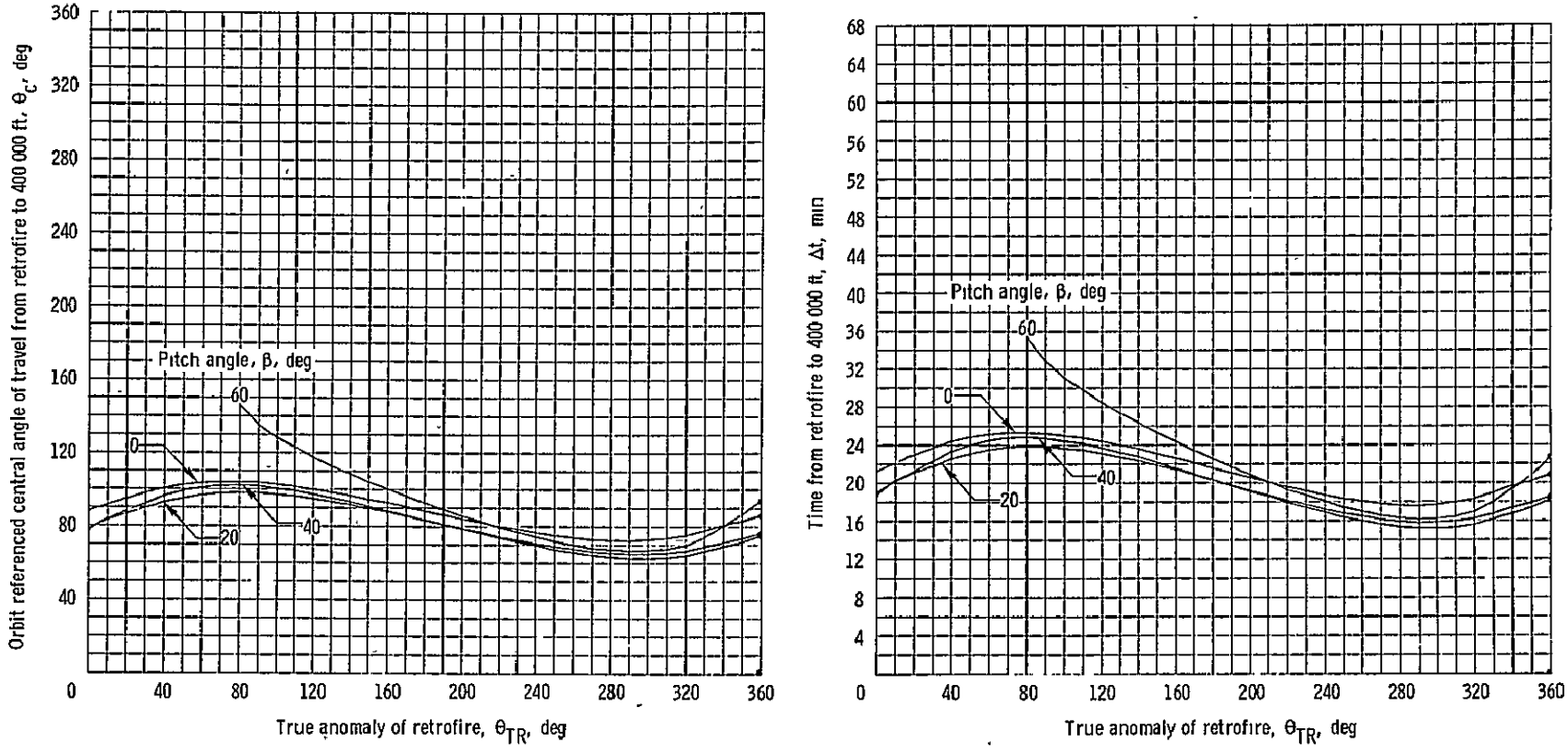
(c) Retrograde ΔV = 500 fps.

Figure 4. - Continued.



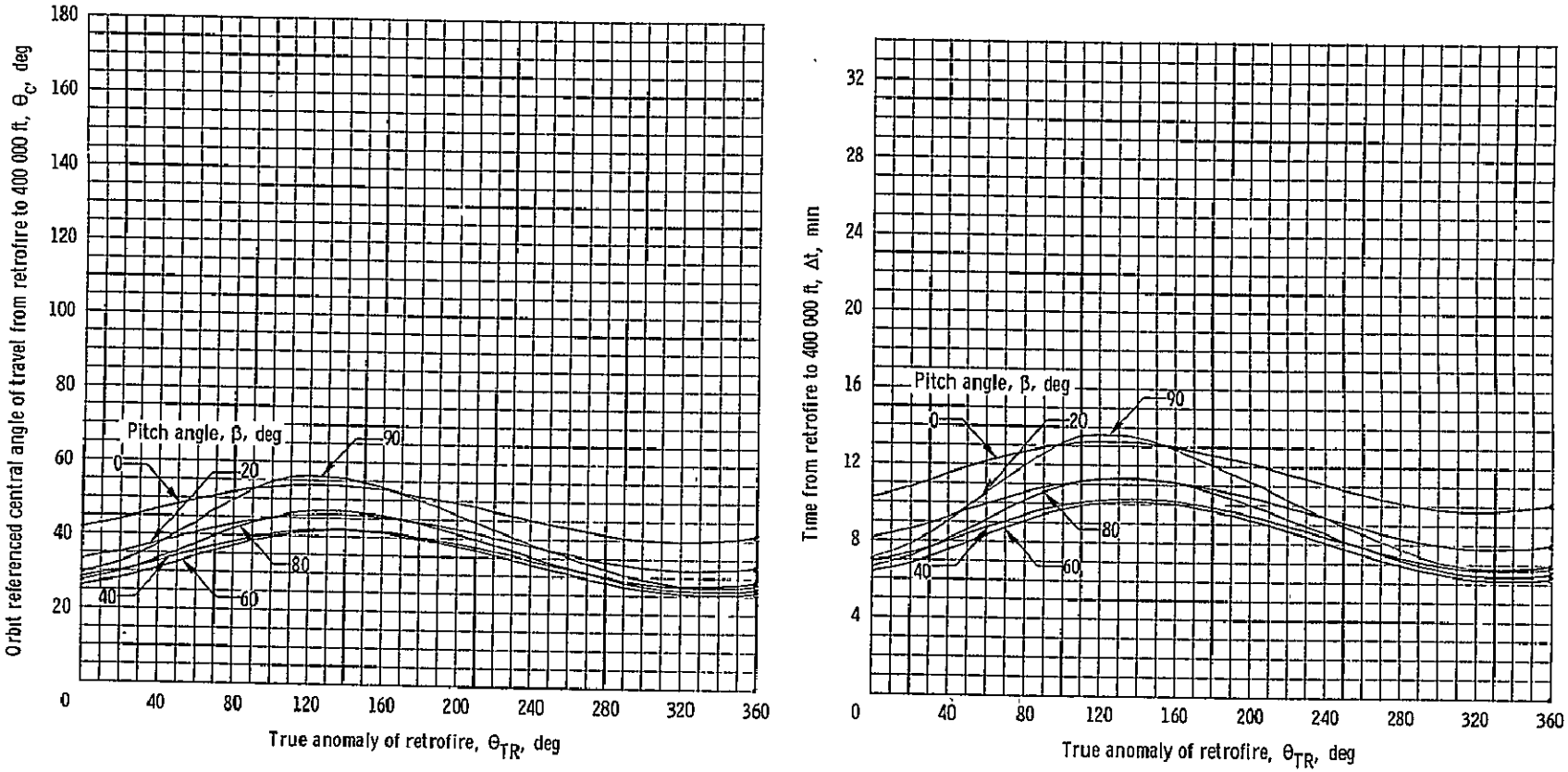
(d) Retrograde ΔV = 700 fps.

Figure 4. - Concluded.



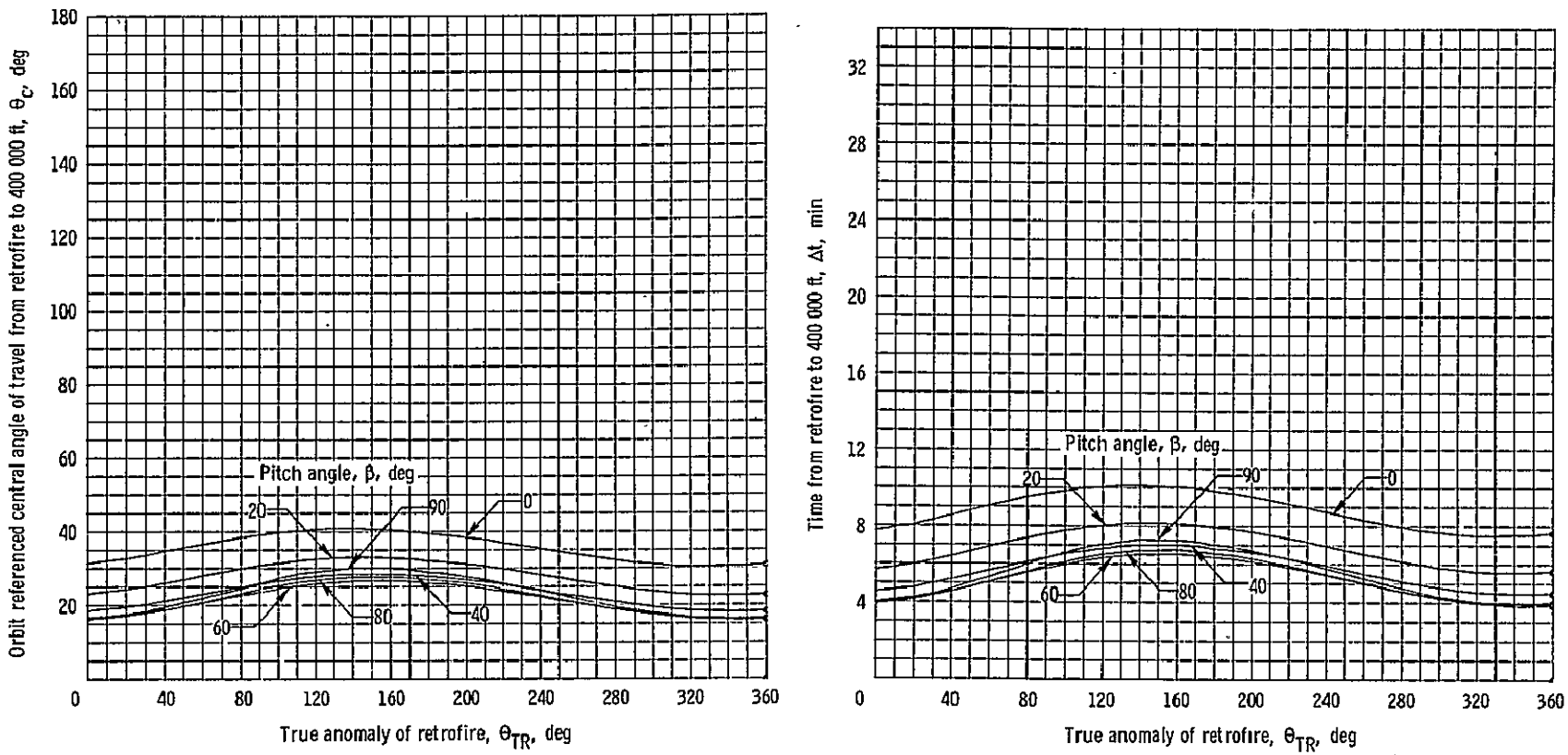
(a) Retrograde ΔV = 100 fps.

Figure 5.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 100 nautical miles and h_p = 85 nautical miles.



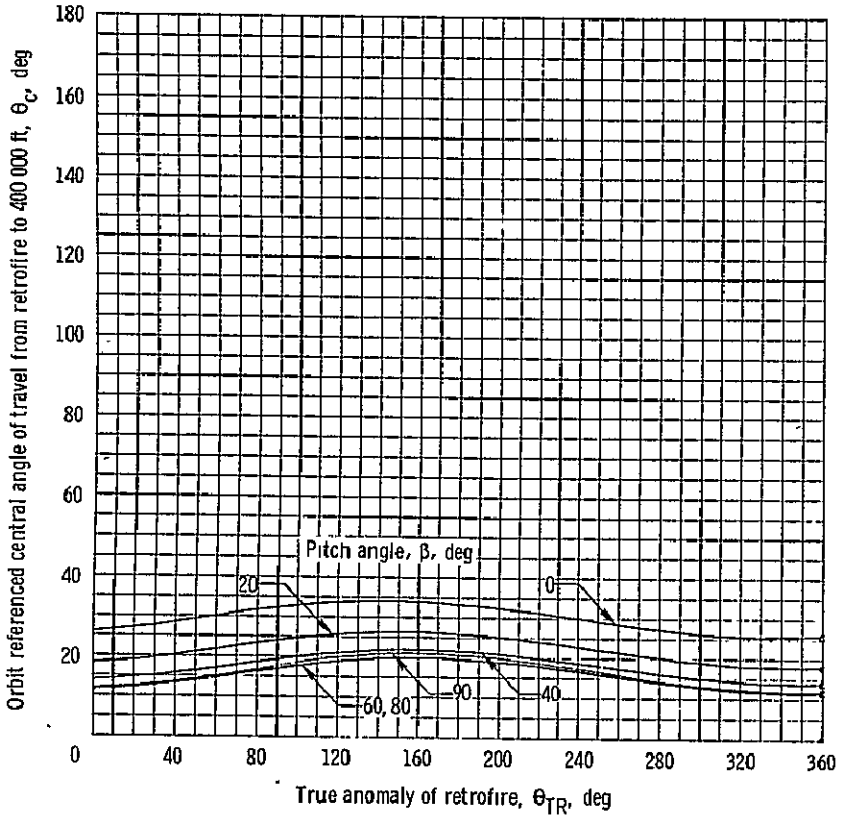
(b) Retrograde ΔV = 300 fps.

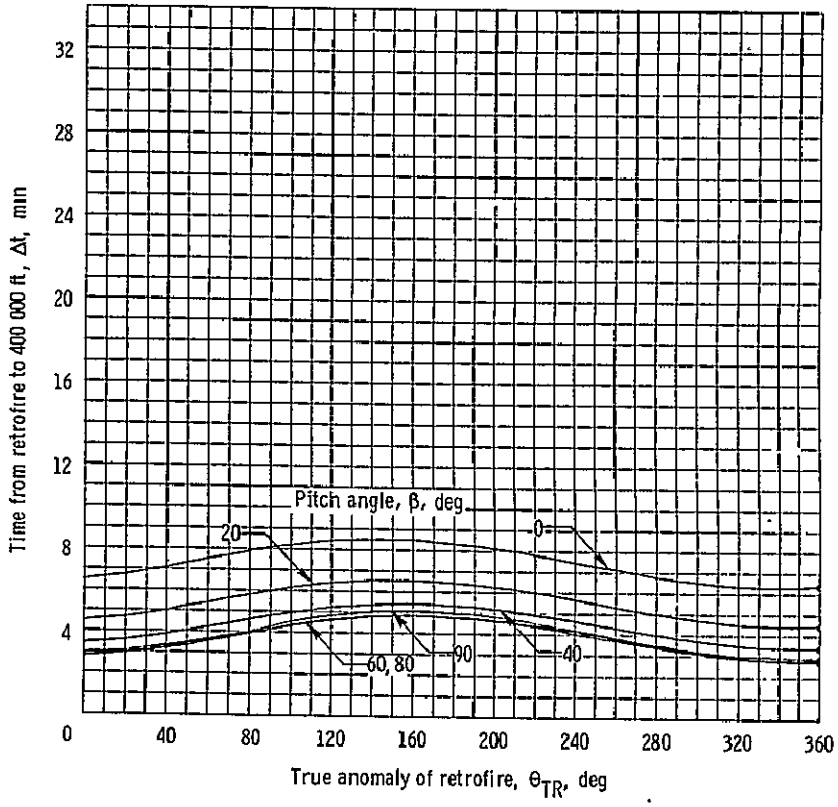
Figure 5. - Continued.



(c) Retrograde ΔV = 500 fps.

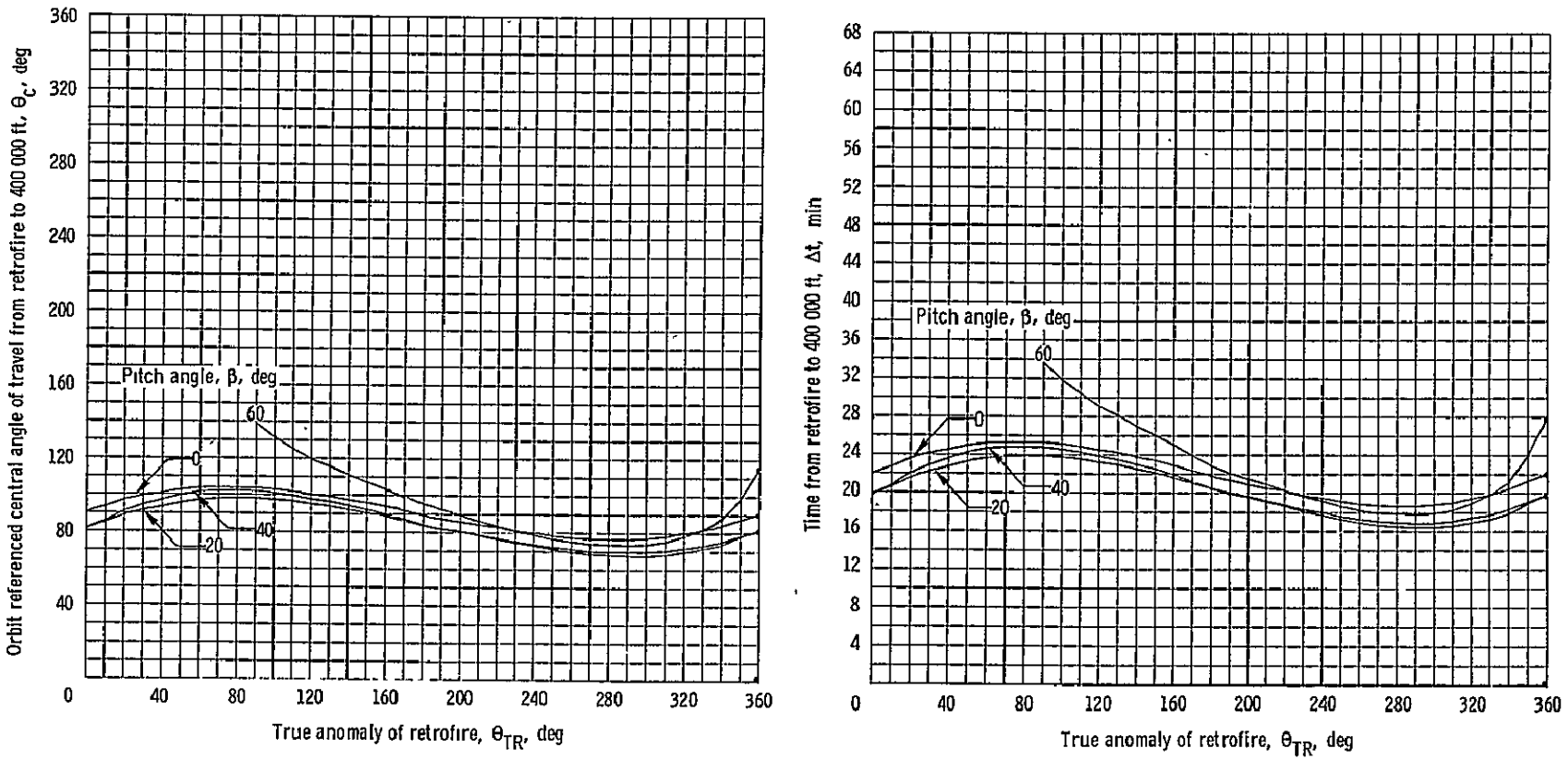
Figure 5. - Continued.





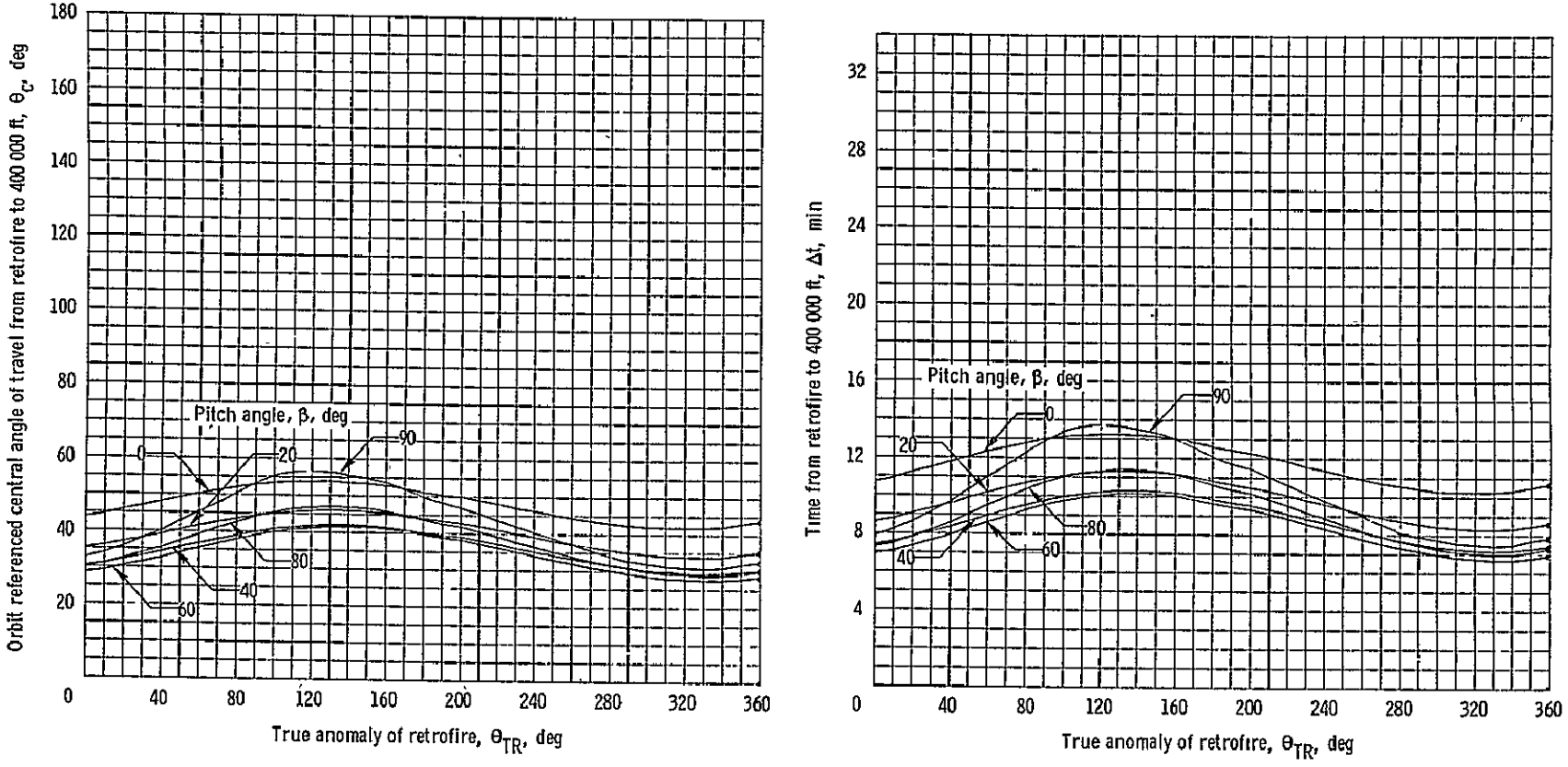
(d) Retrograde ΔV = 700 fps.

Figure 5. - Concluded.



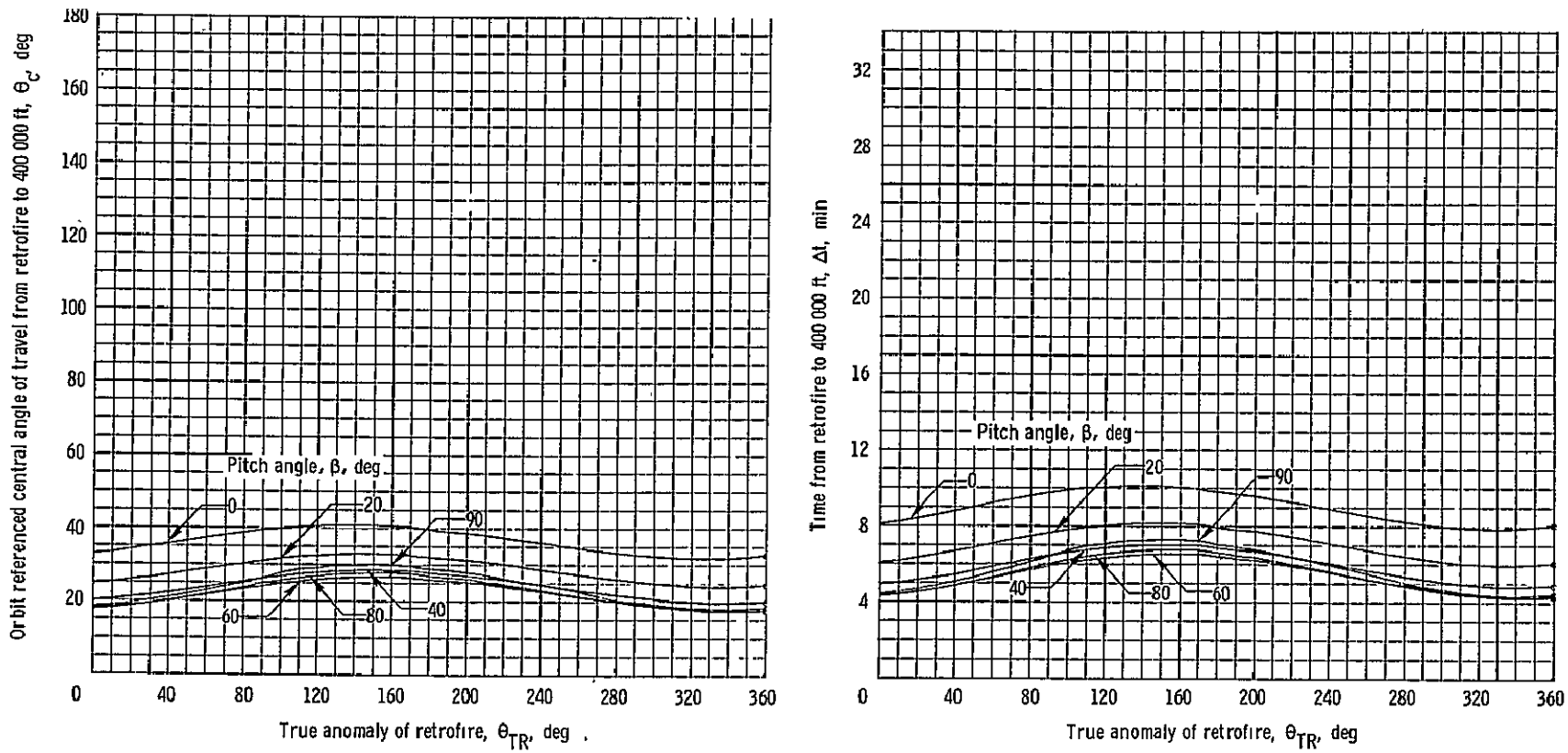
(a) Retrograde ΔV = 100 fps.

Figure 6.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 100 nautical miles and h_p = 87 nautical miles.



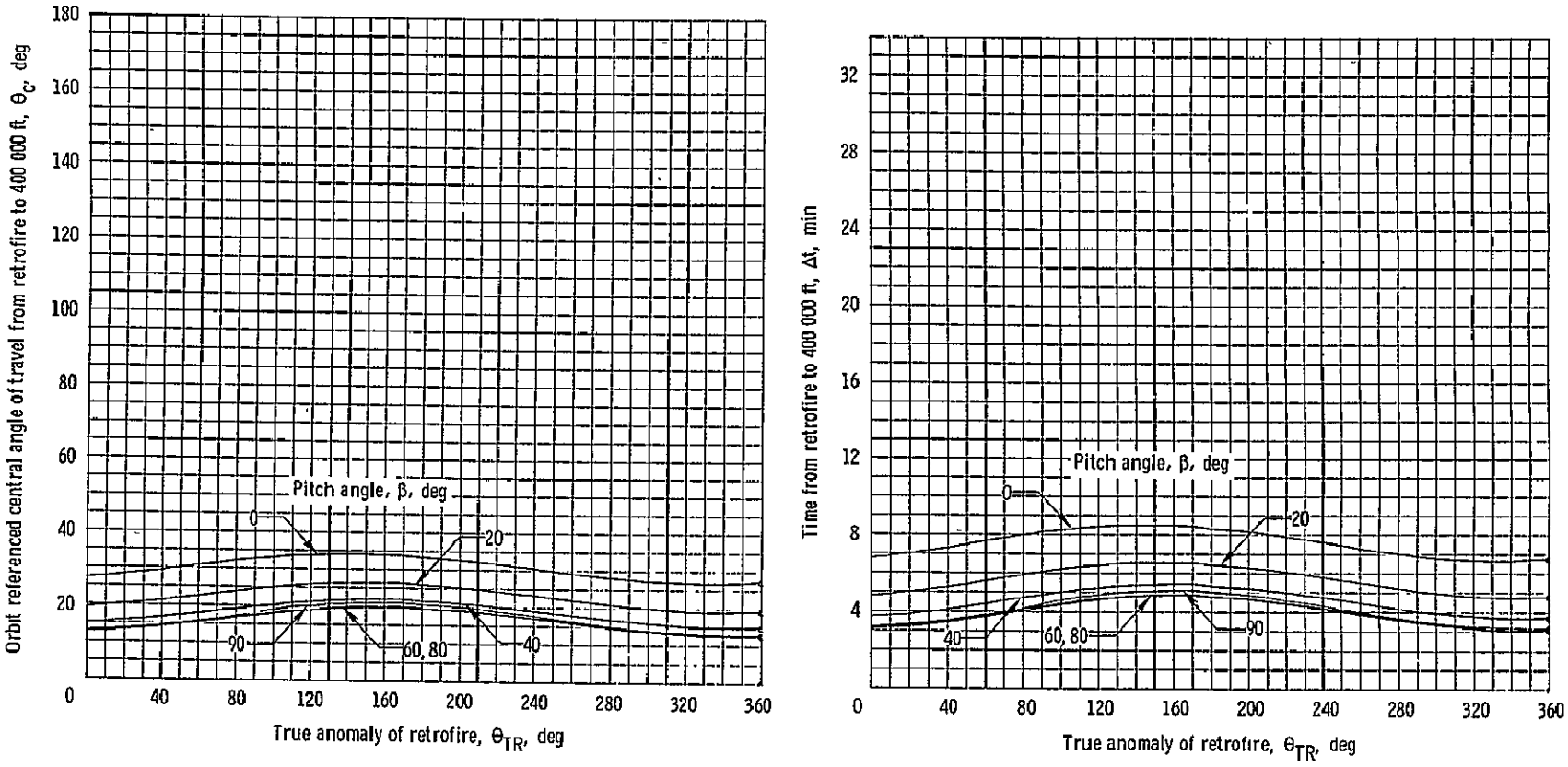
(b) Retrograde ΔV = 300 fps.

Figure 6.- Continued.



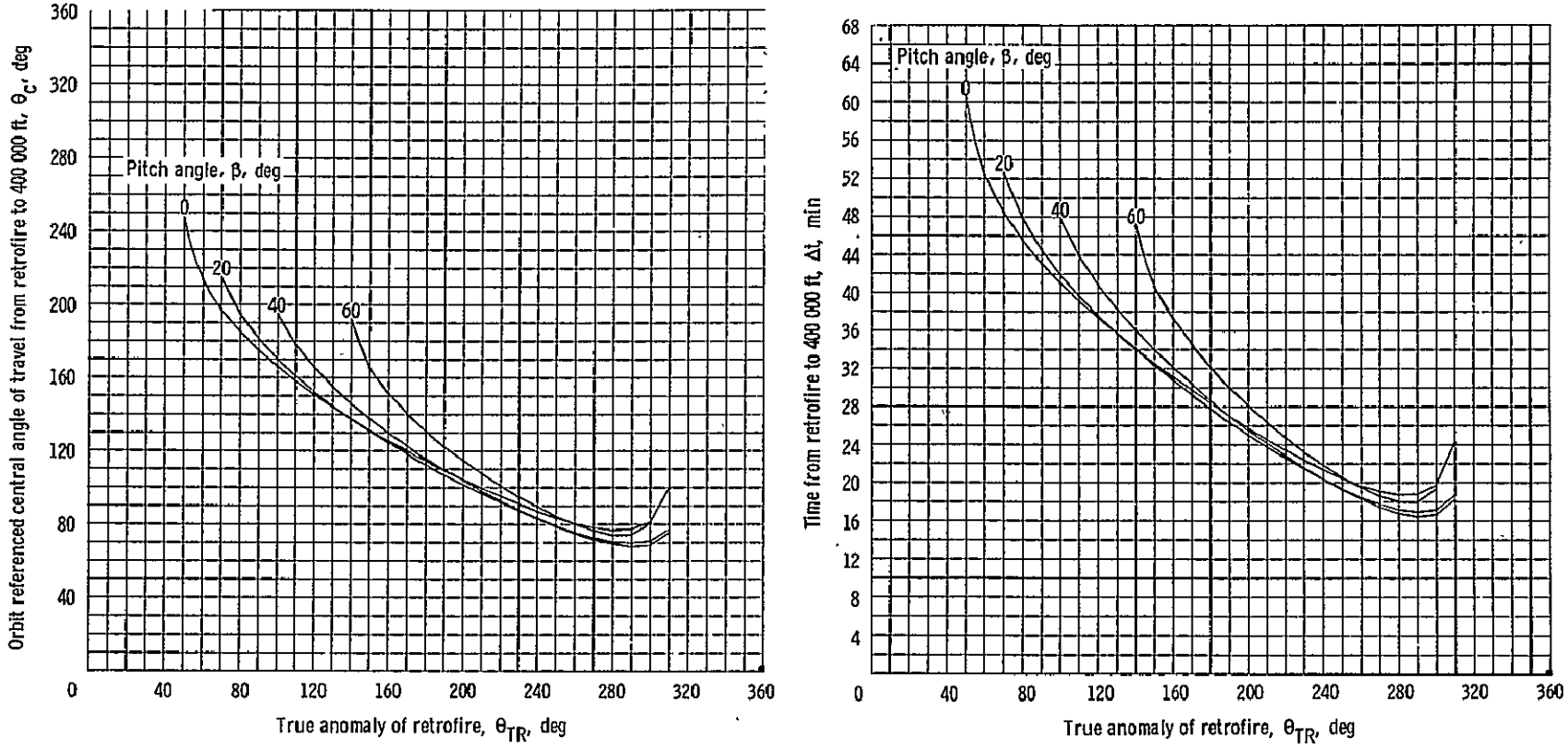
(c) Retrograde ΔV = 500 fps.

Figure 6. - Continued.



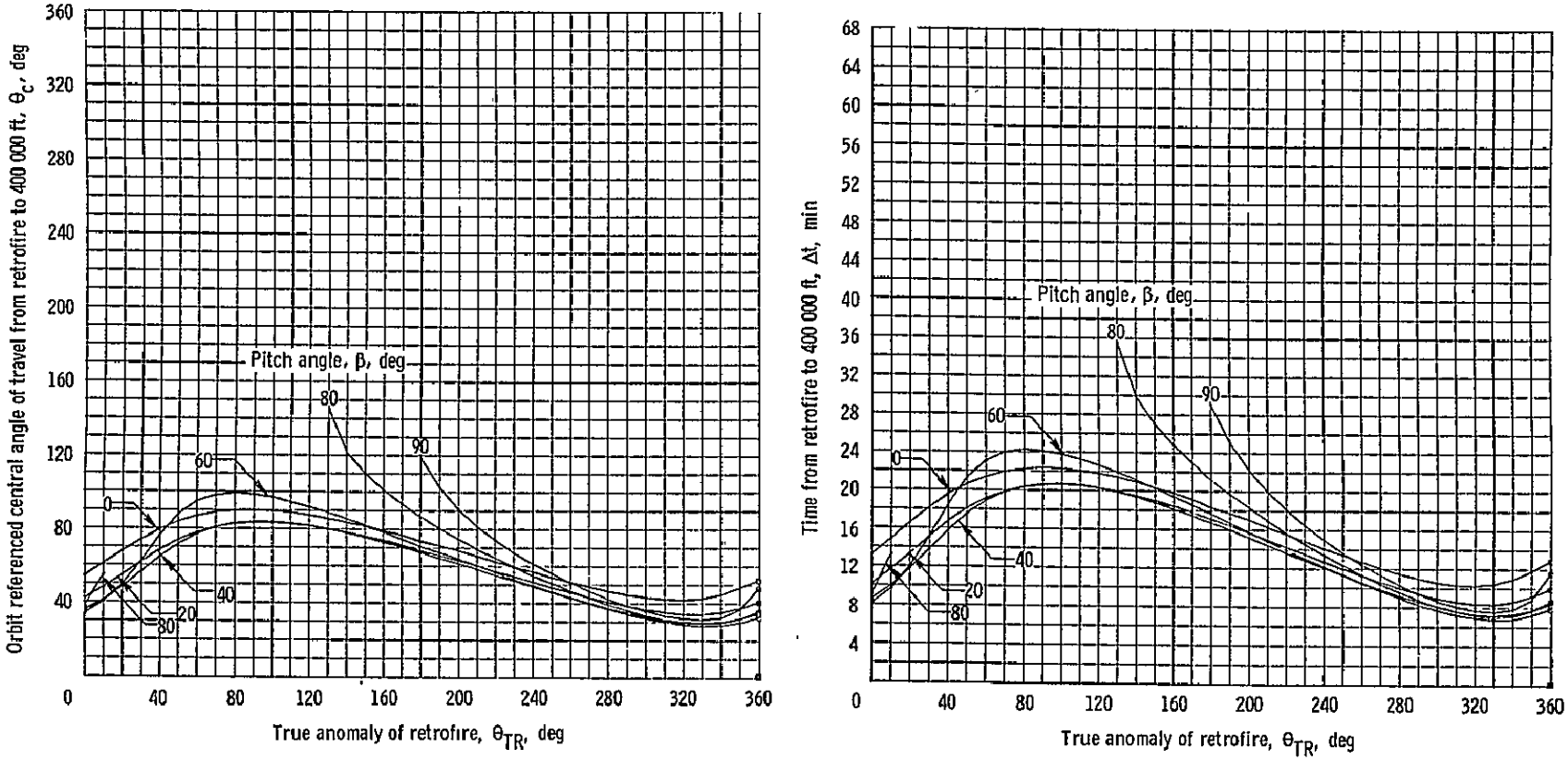
(d) Retrograde ΔV = 700 fps.

Figure 6.- Concluded.



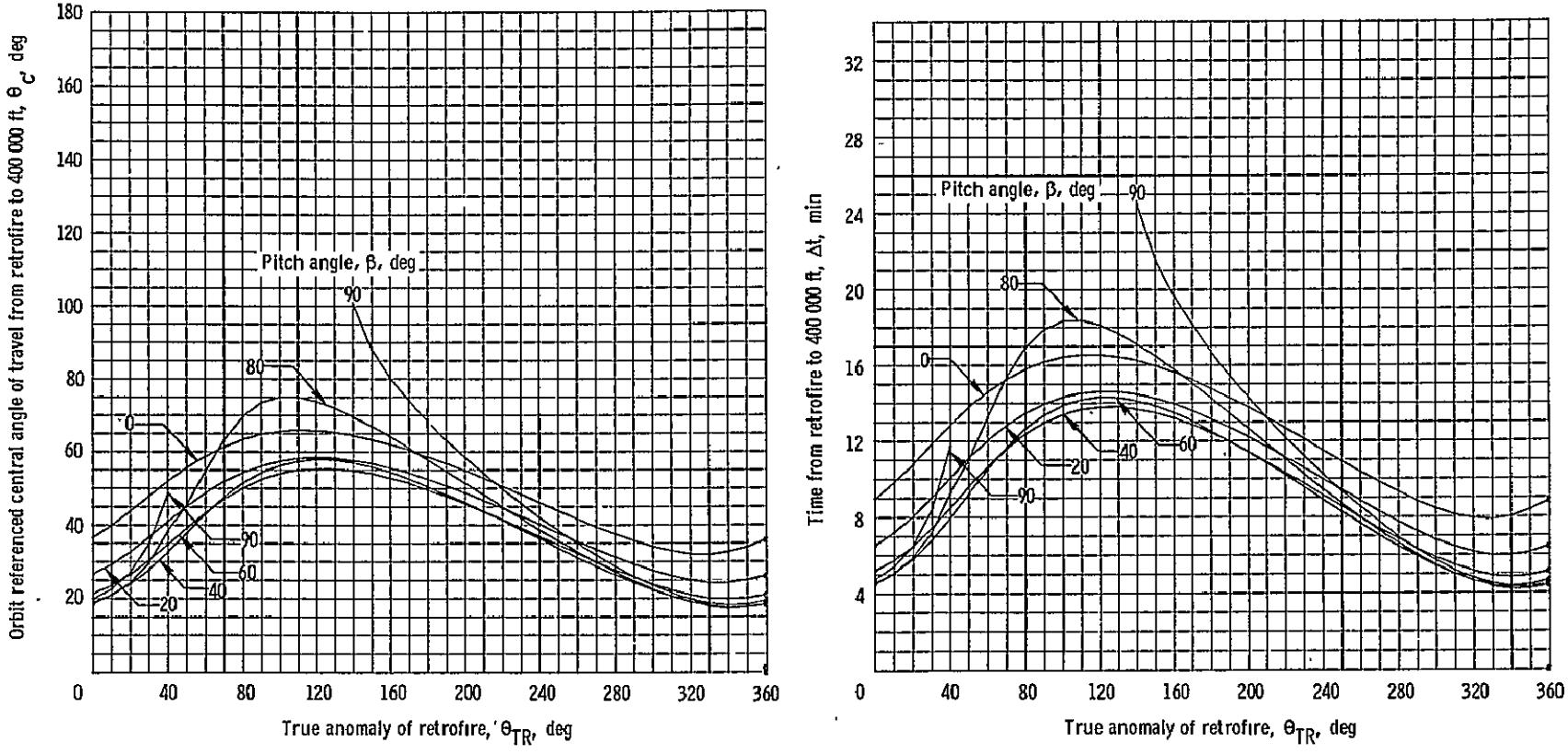
(a) Retrograde ΔV = 100 fps.

Figure 7.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 150 nautical miles and h_p = 87 nautical miles.



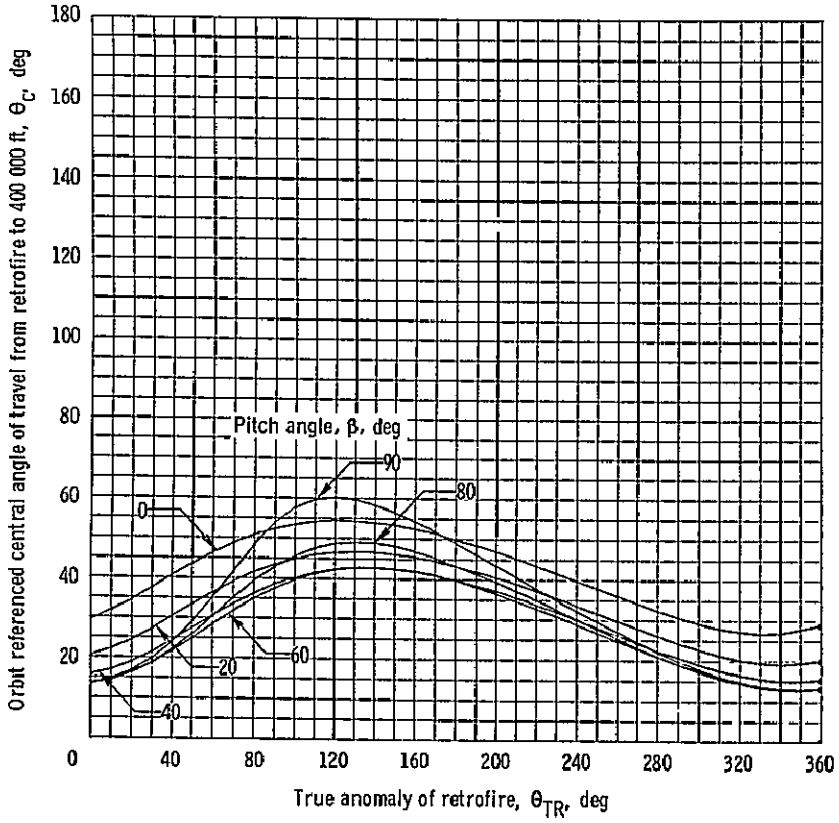
(b) Retrograde ΔV = 300 fps.

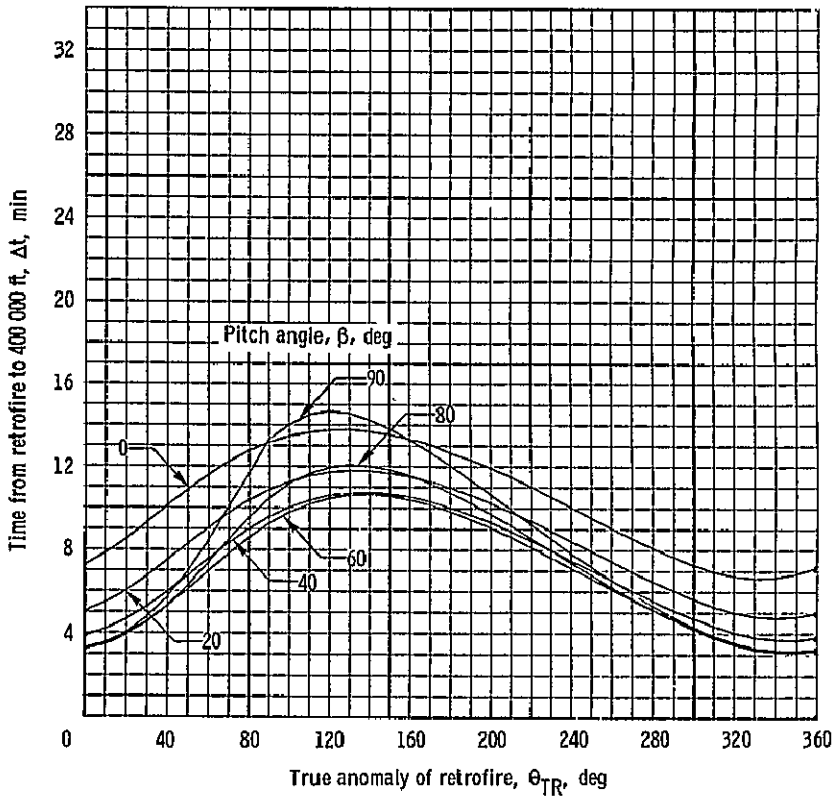
Figure 7. - Continued.



(c) Retrograde ΔV = 500 fps.

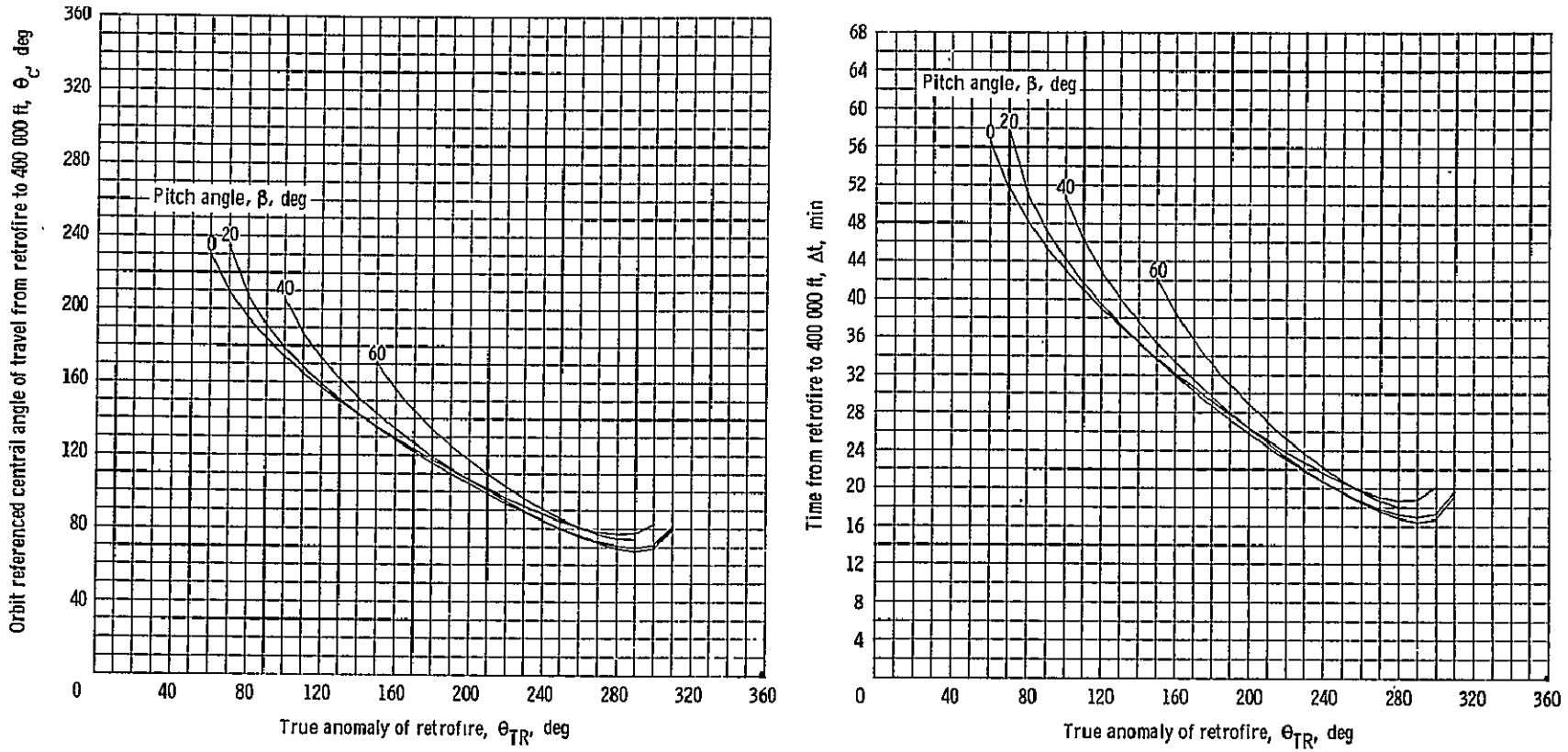
Figure 7. - Continued.





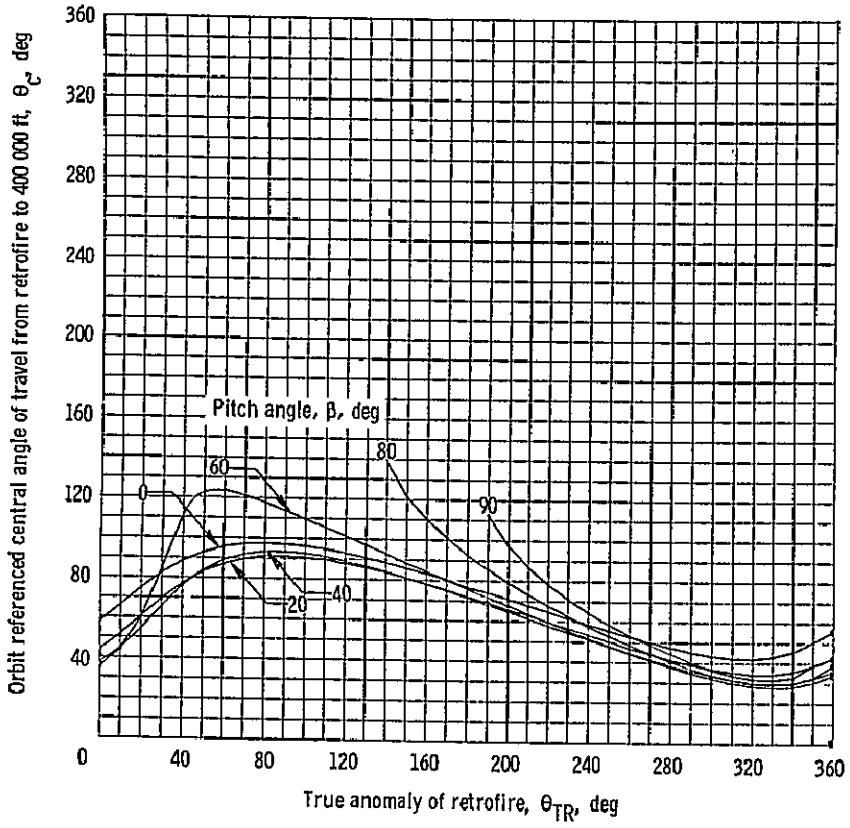
(d) Retrograde ΔV = 700 fps.

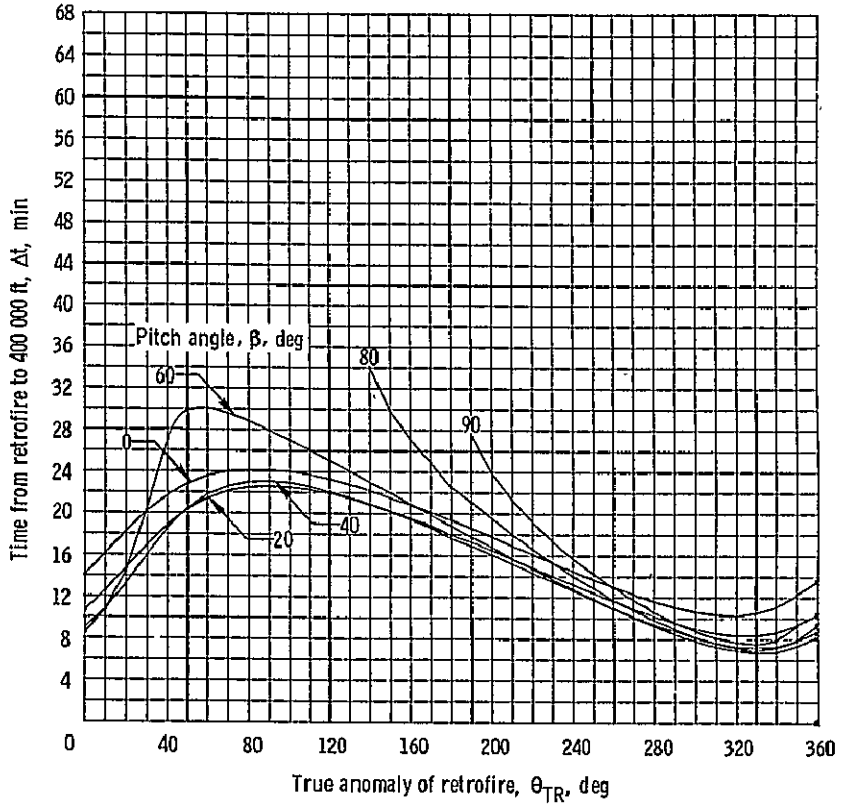
Figure 7. - Concluded.



(a) Retrograde ΔV = 100 fps.

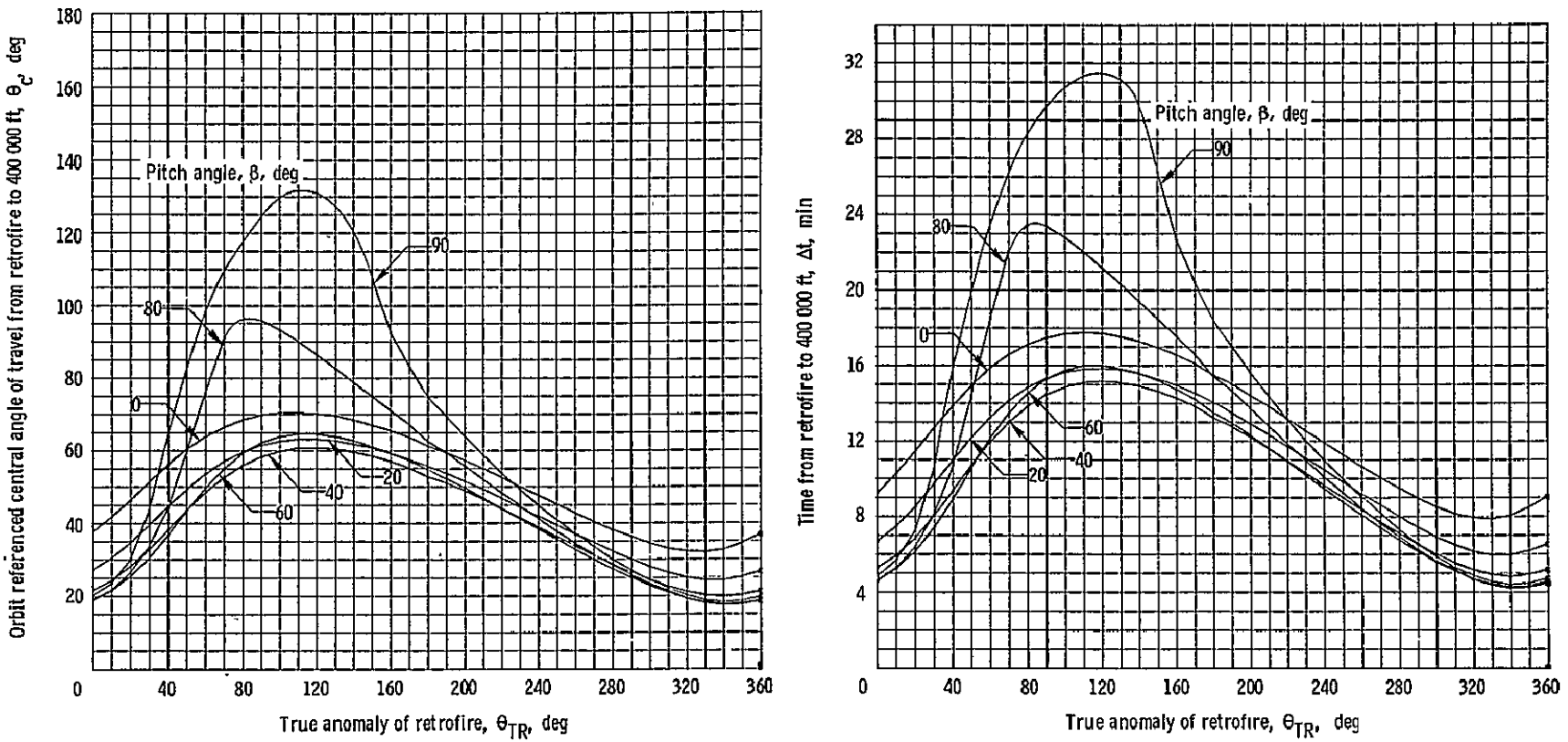
Figure 8.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 161 nautical miles and h_p = 87 nautical miles.





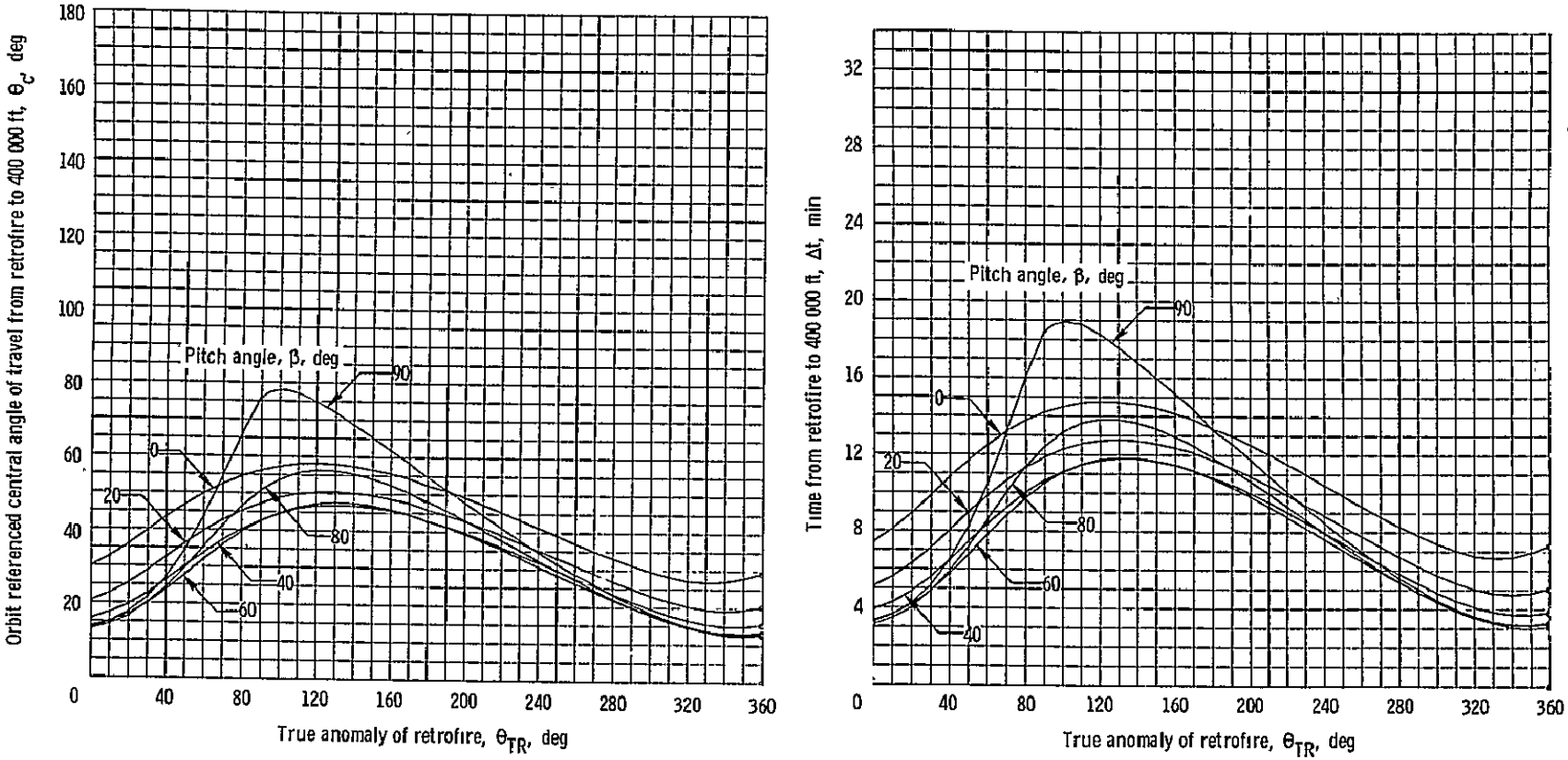
(b) Retrograde ΔV = 300 fps.

Figure 8. - Continued.



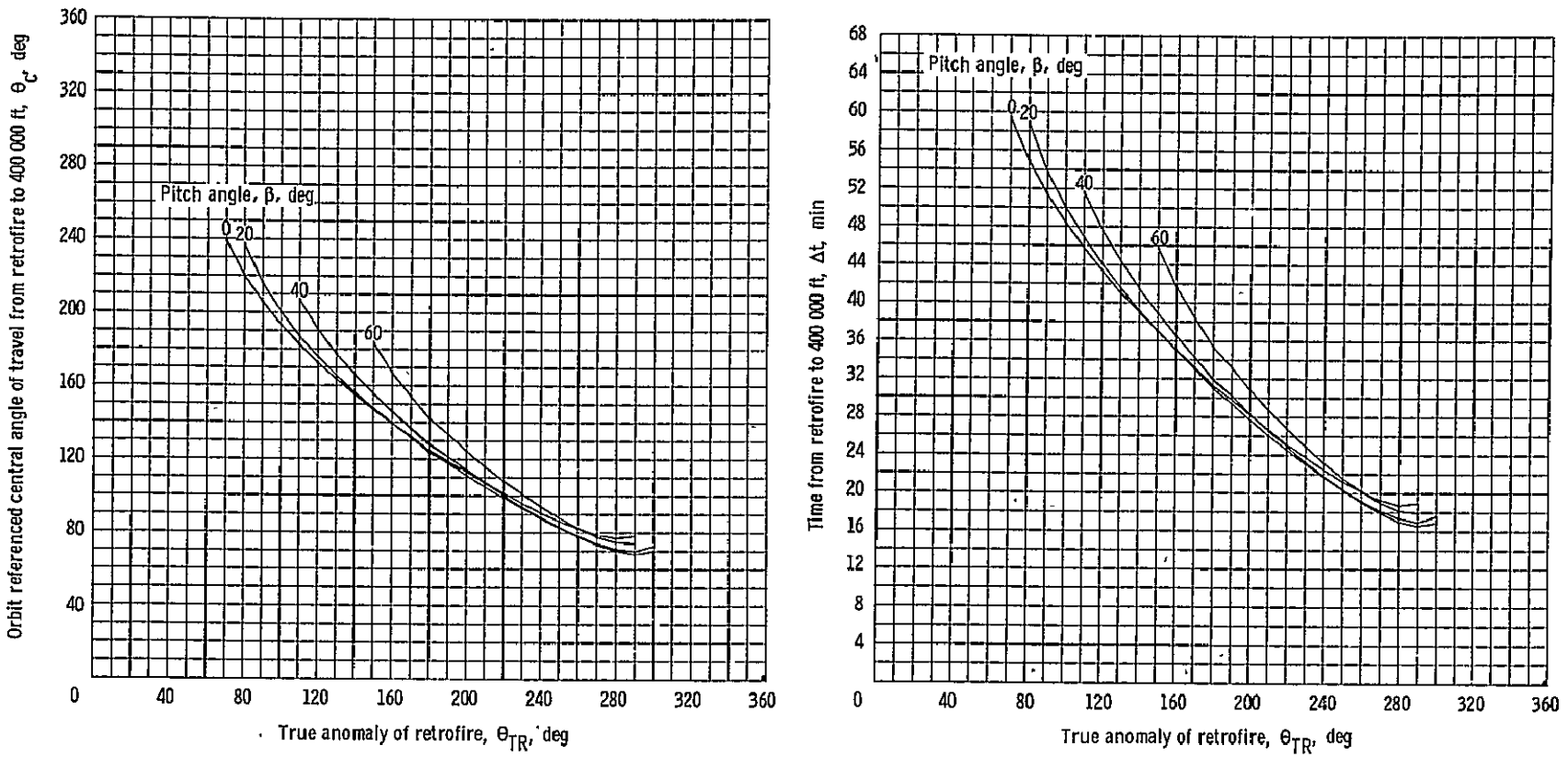
(c) Retrograde ΔV = 500 fps.

Figure 8. - Continued.



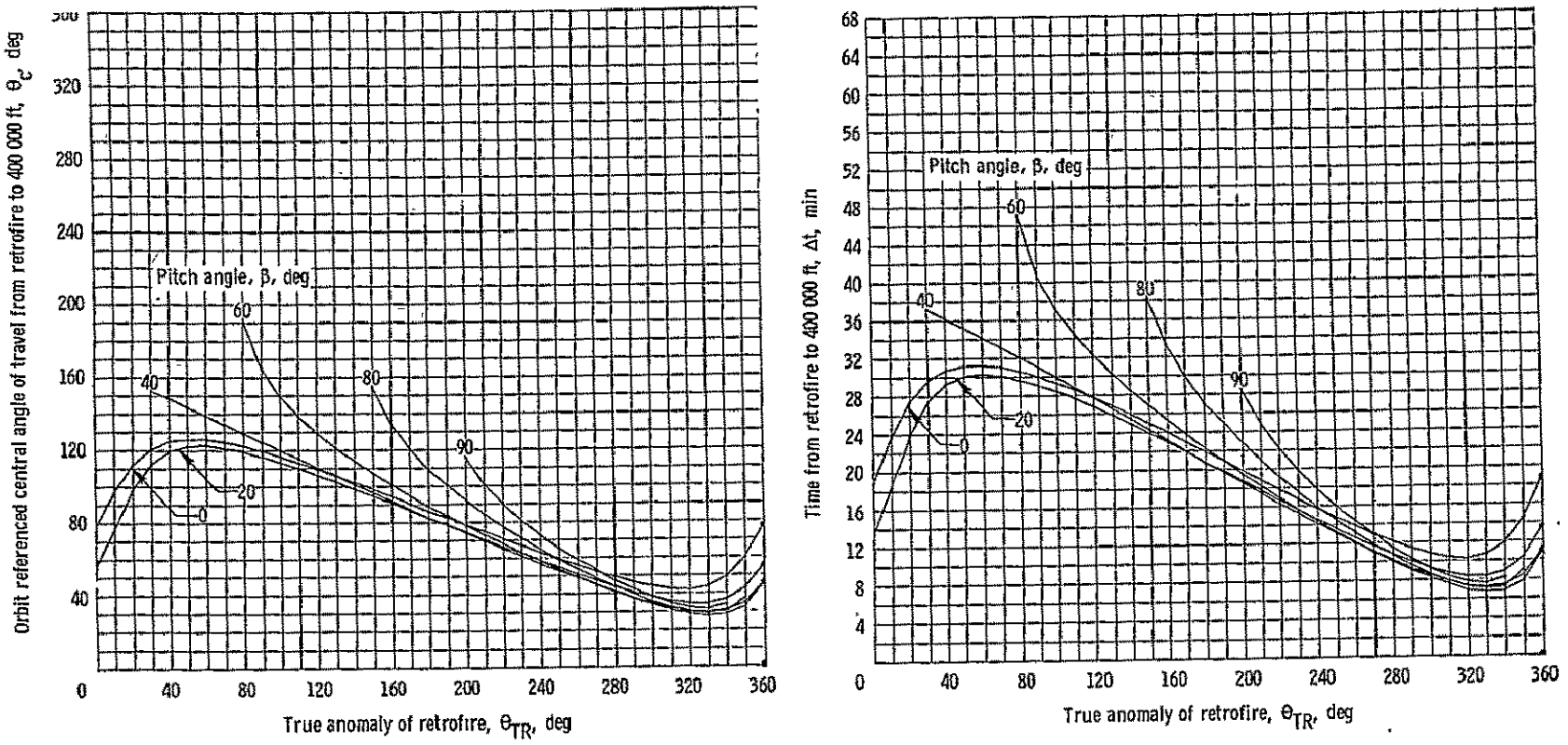
(d) Retrograde ΔV = 700 fps.

Figure 8. - Concluded.



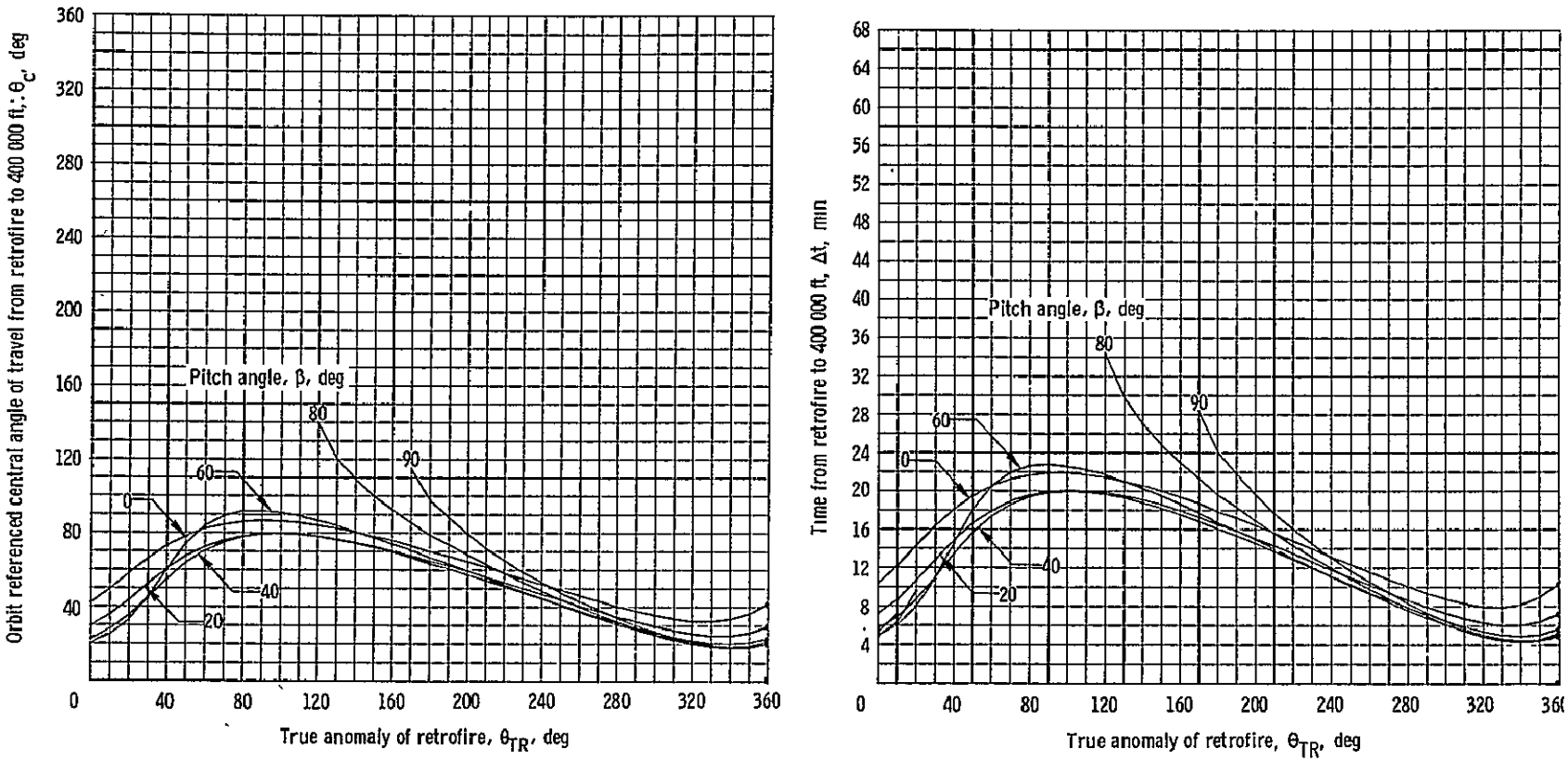
(a) Retrograde ΔV = 100 fps.

Figure 9.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 200 nautical miles and h_p = 87 nautical miles.



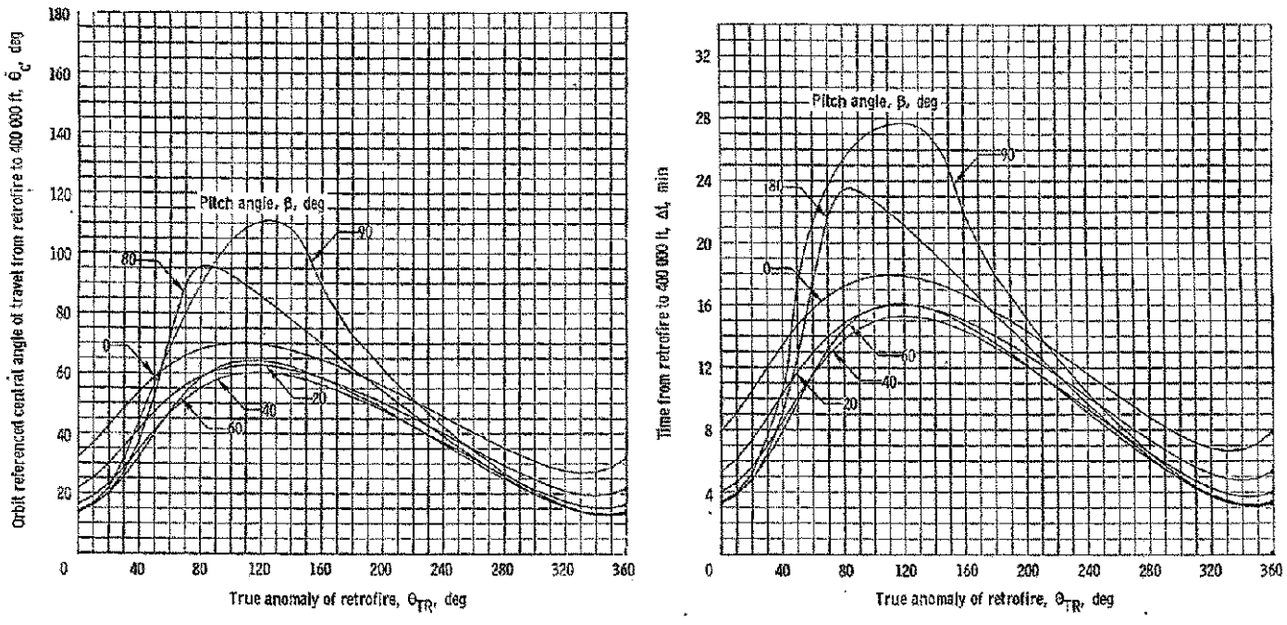
(b) Retrograde ΔV = 300 fps.

Figure 9. - Continued.



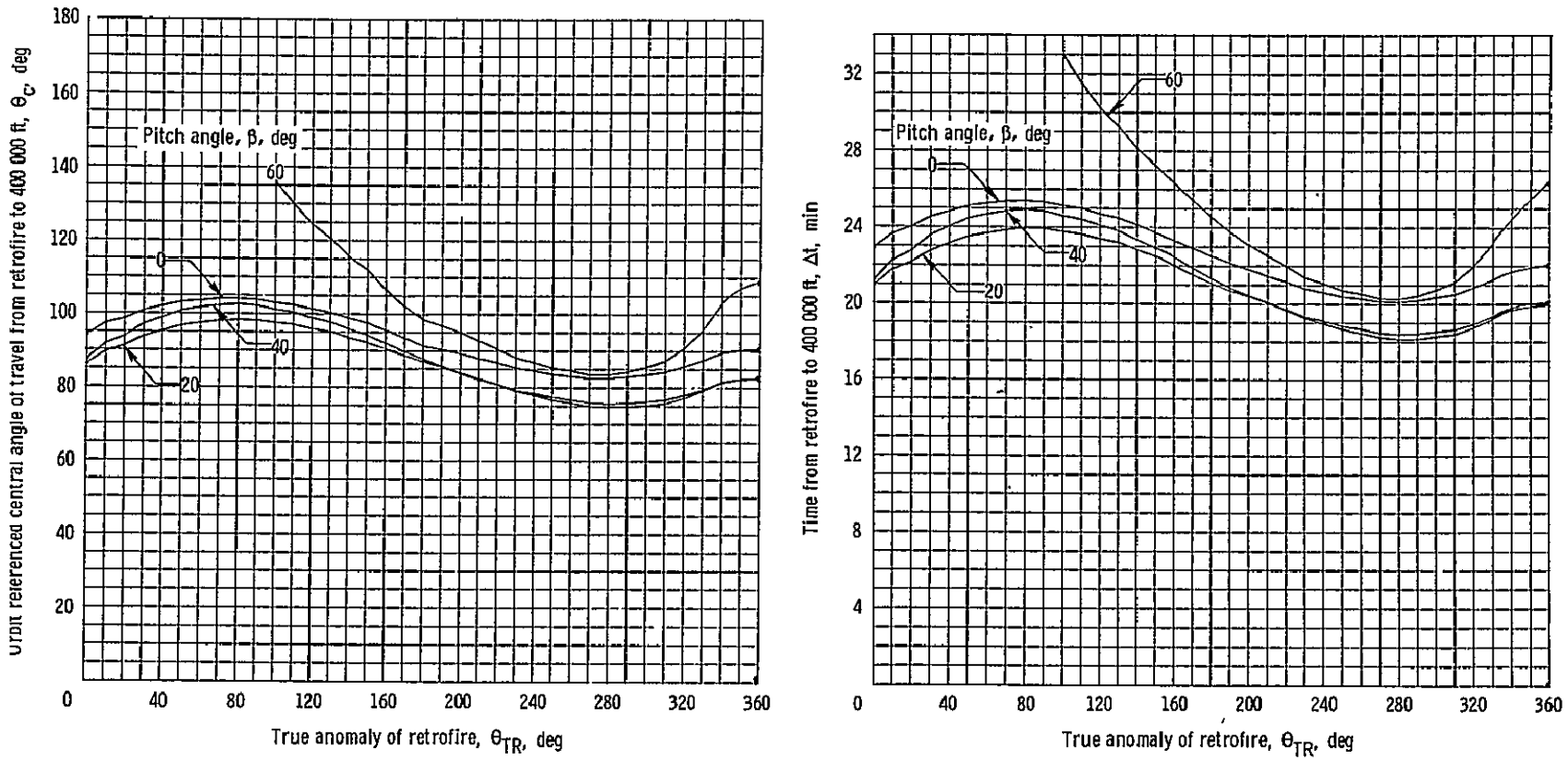
(c) Retrograde ΔV = 500 fps.

Figure 9. – Continued.



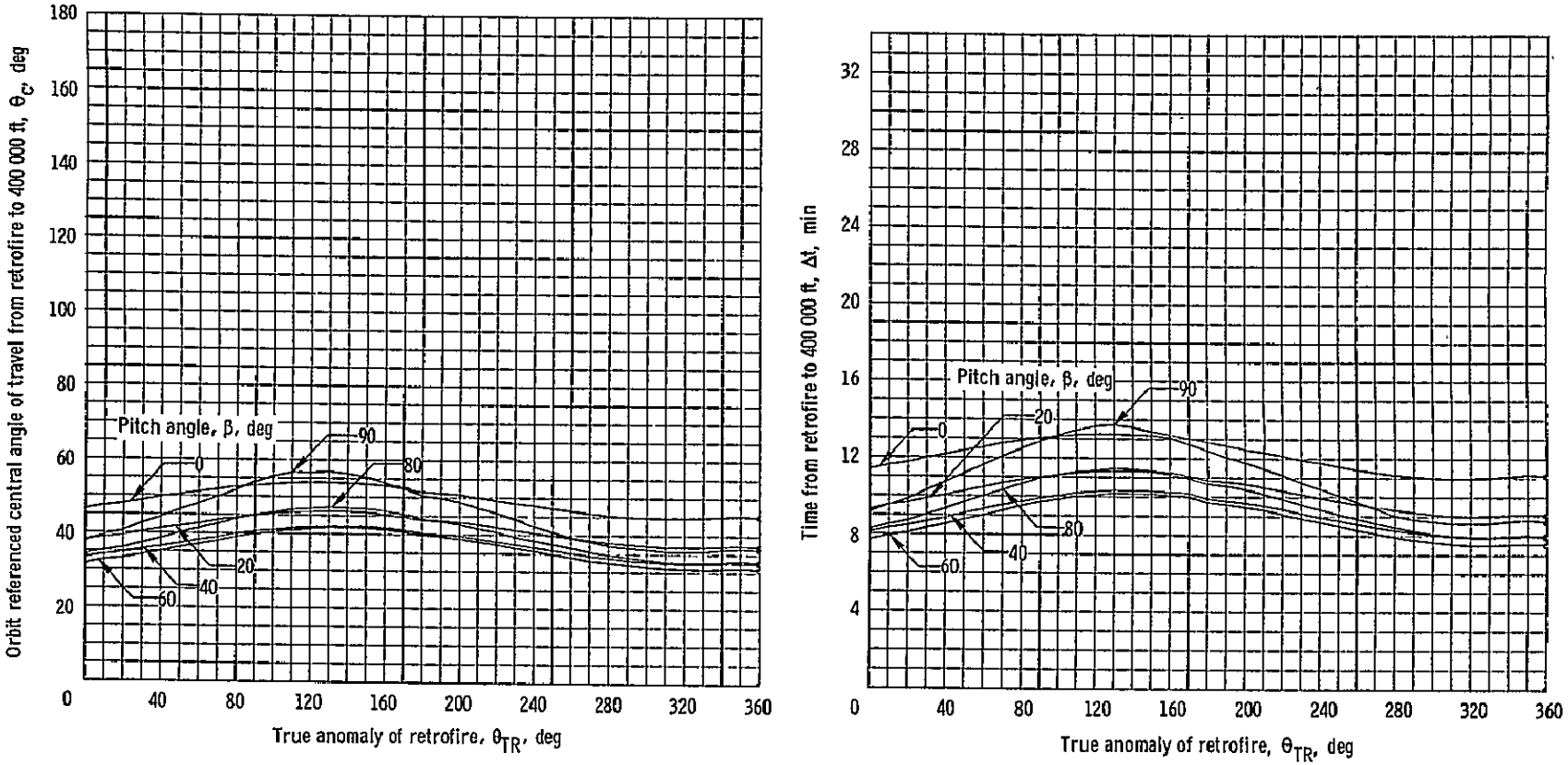
(d) Retrograde ΔV = 700 fps.

Figure 9. - Concluded.



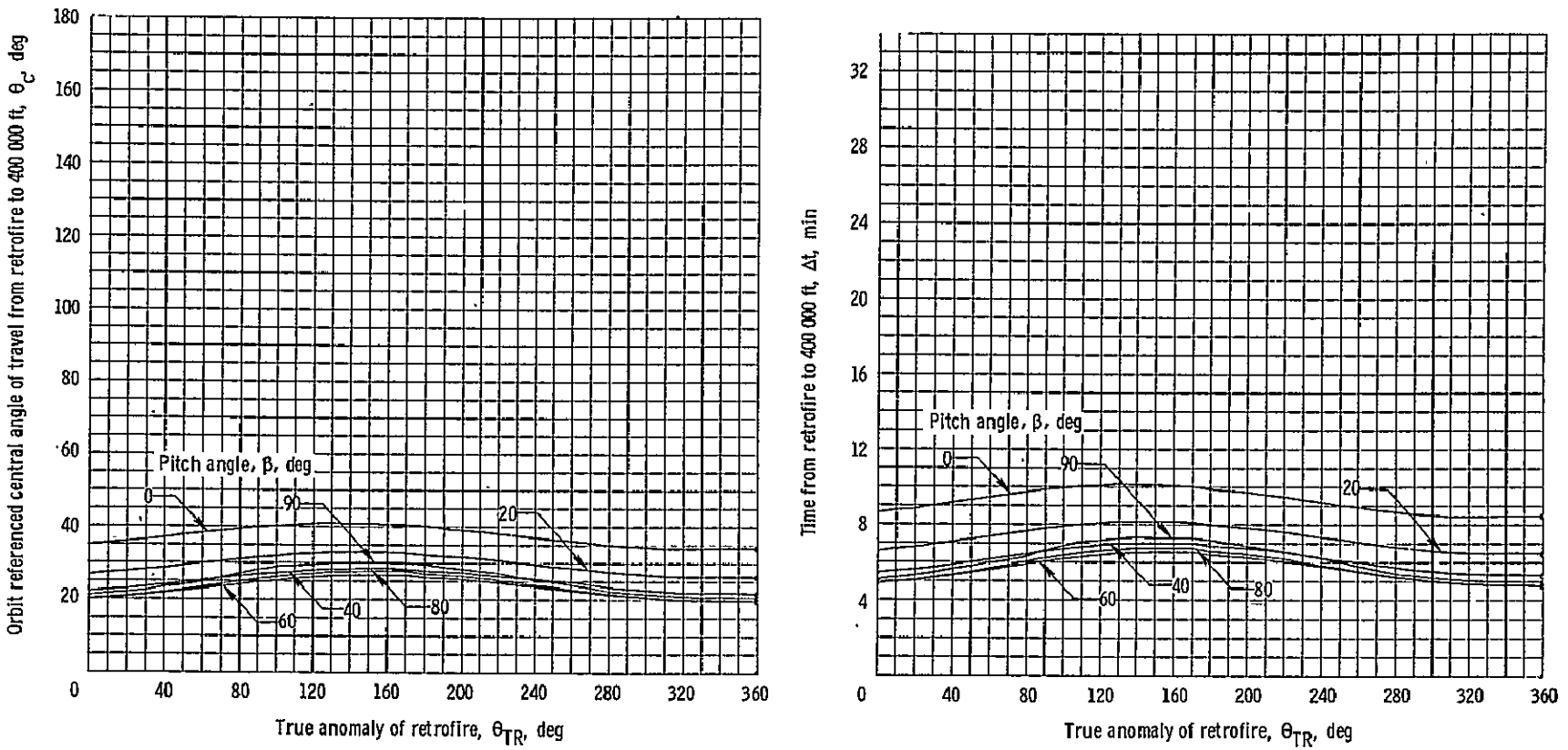
(a Retrograde ΔV = 100 fps.

Figure 10.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 100 nautical miles and h_p = 90 nautical miles.



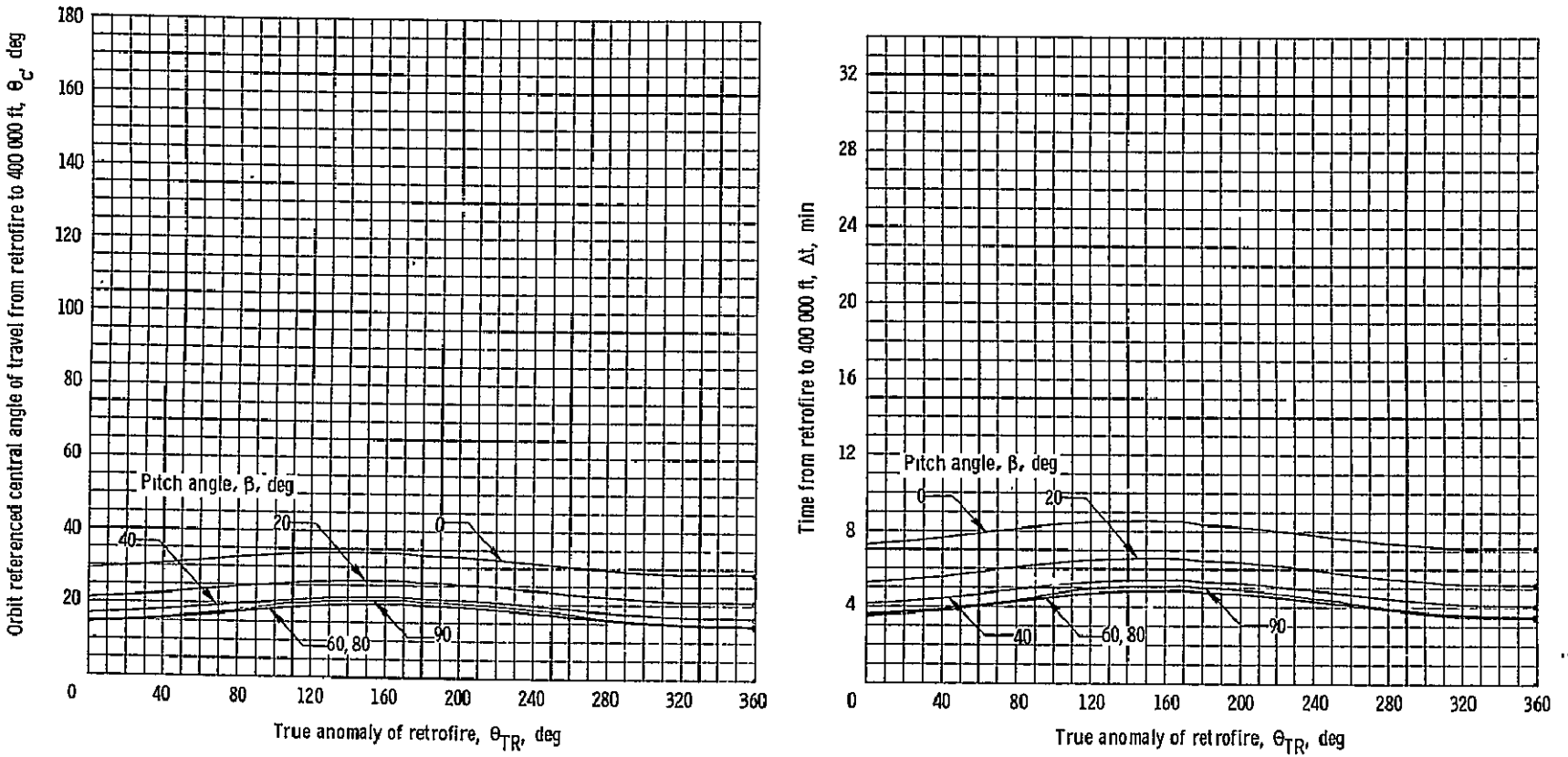
(b) Retrograde ΔV = 300 fps.

Figure 10. - Continued.



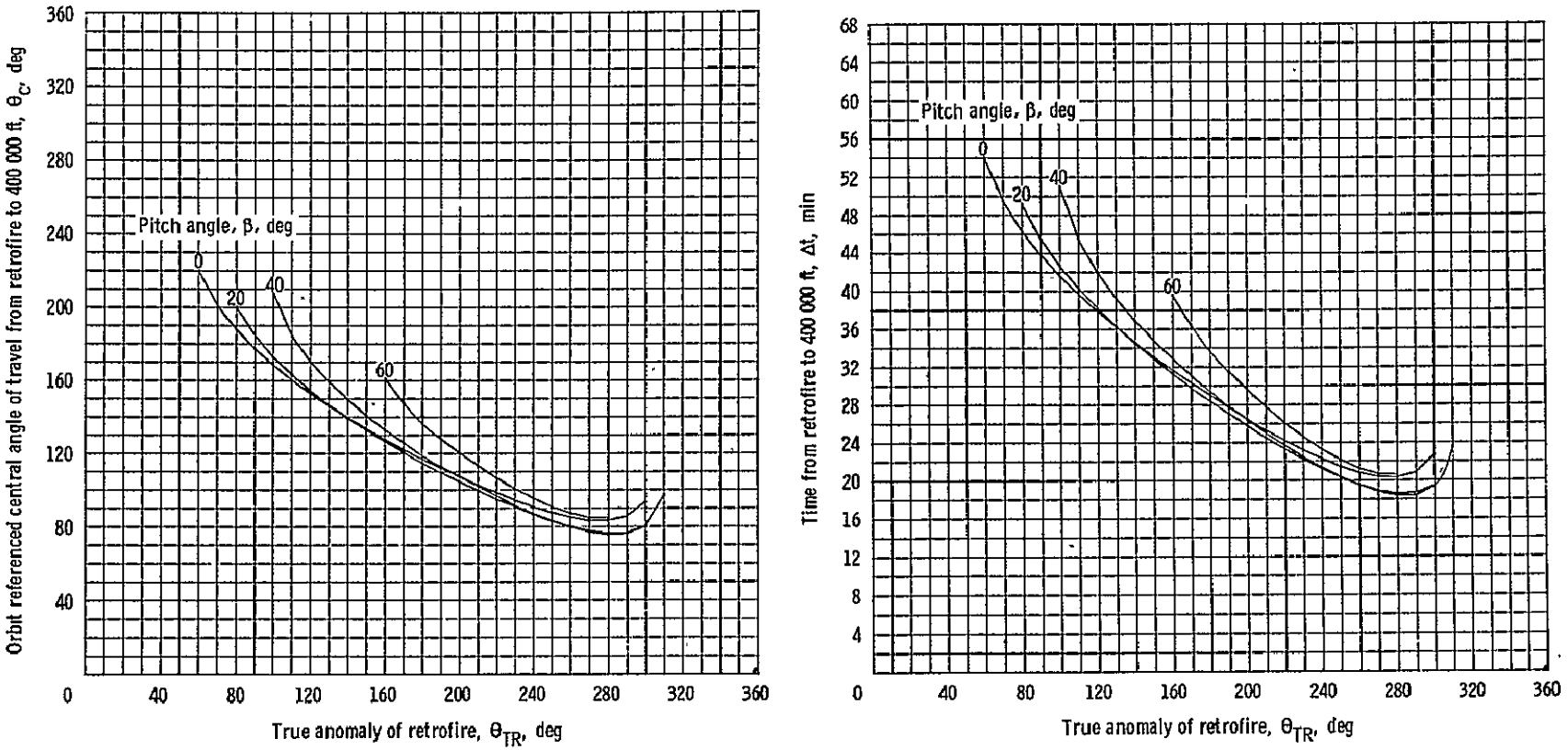
(c) Retrograde ΔV = 500 fps.

Figure 10. - Continued.



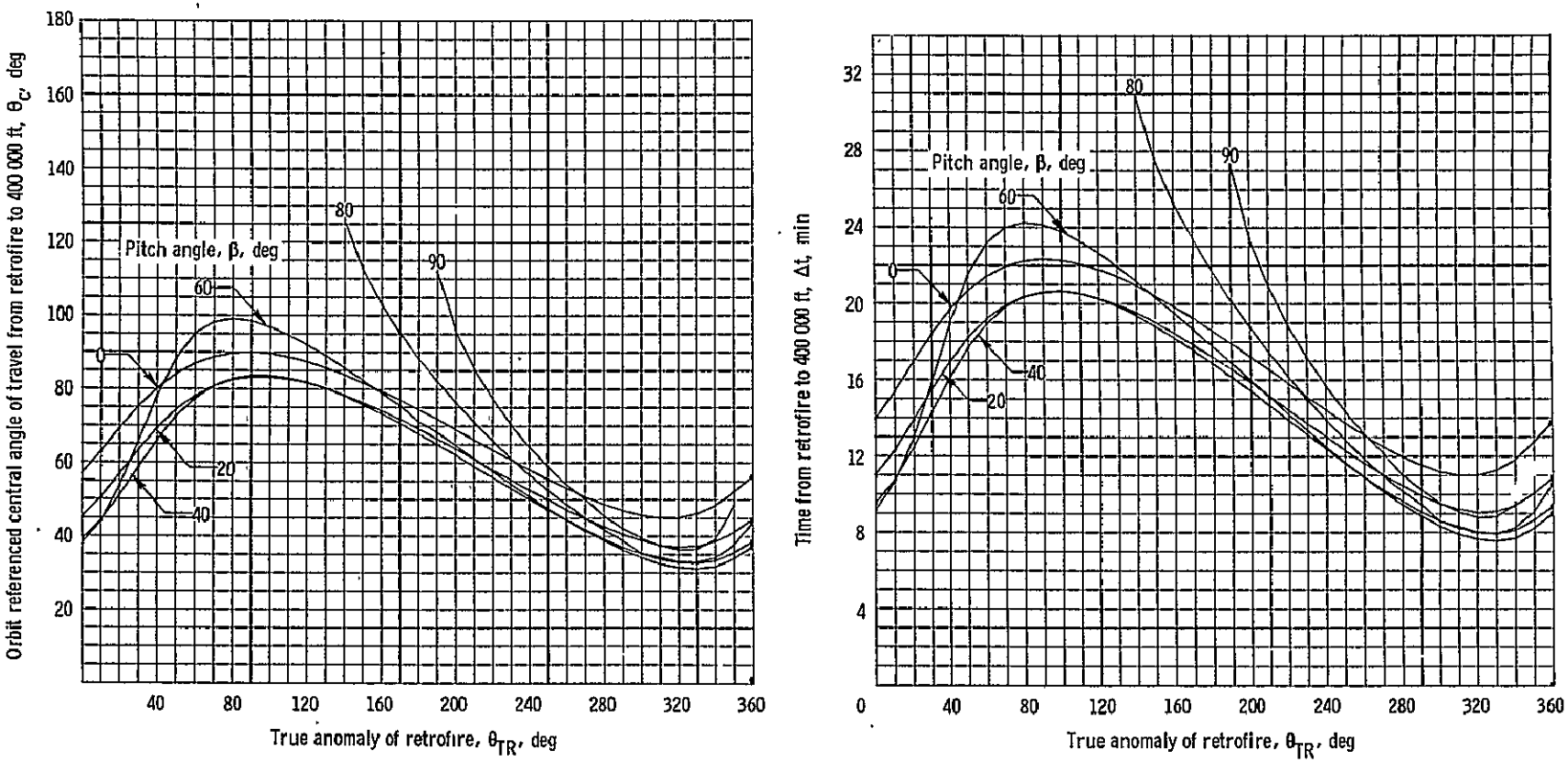
(d) Retrograde ΔV = 700 fps.

Figure 10.- Concluded.



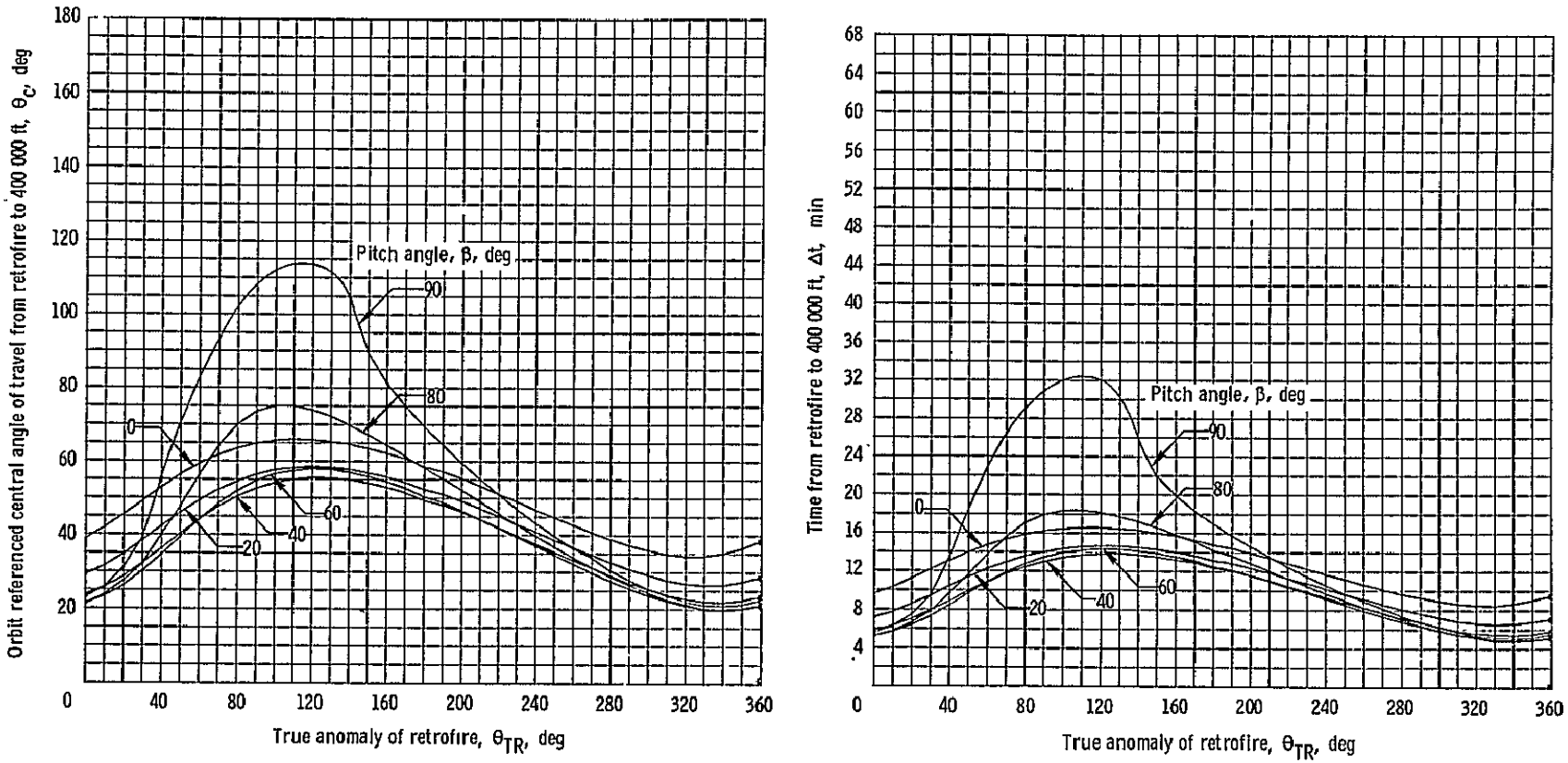
(a) Retrograde ΔV = 100 fps.

Figure 11.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 150 nautical miles and h_p = 90 nautical miles.



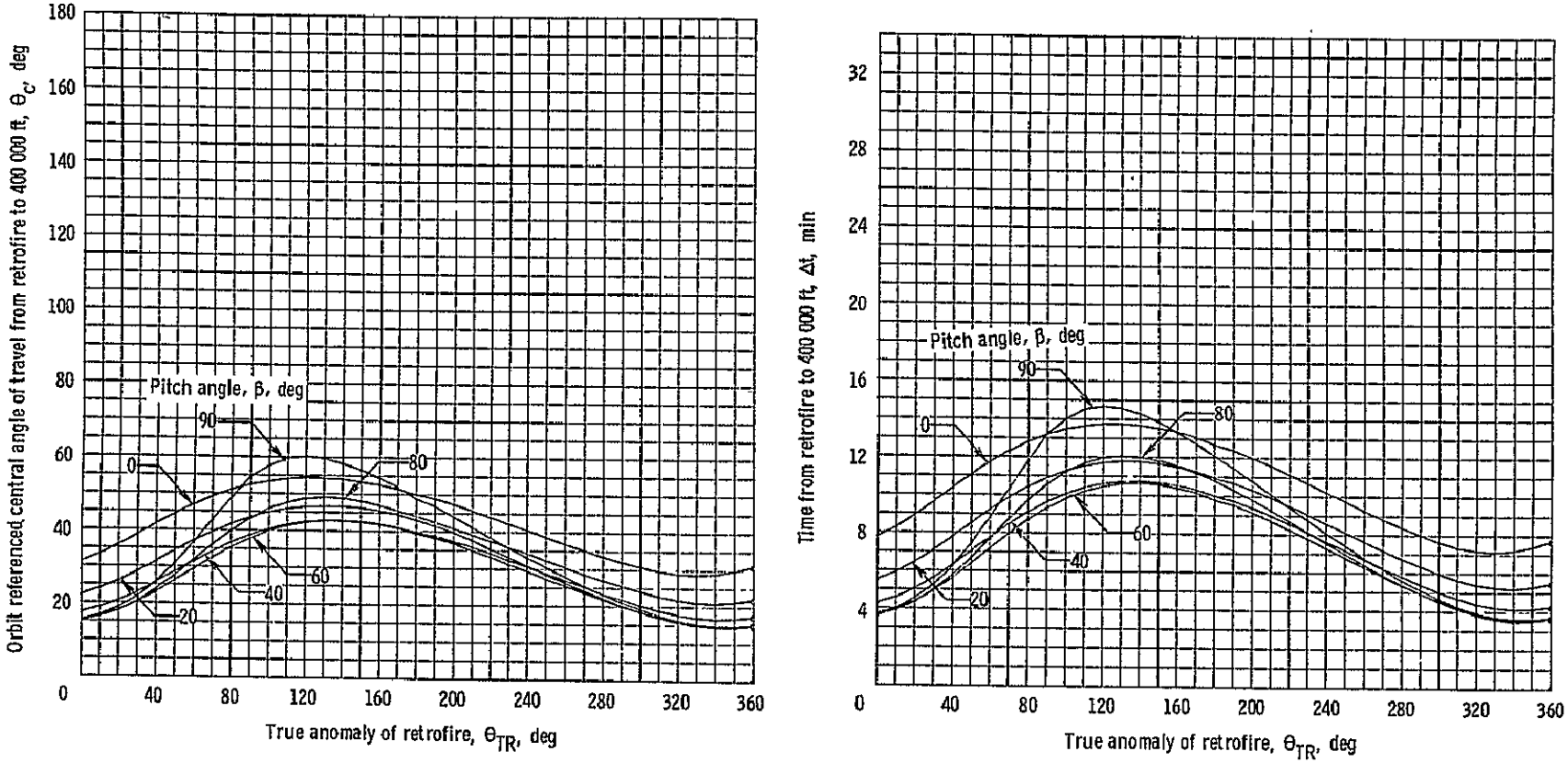
(b) Retrograde ΔV = 300 fps.

Figure 11. - Continued.



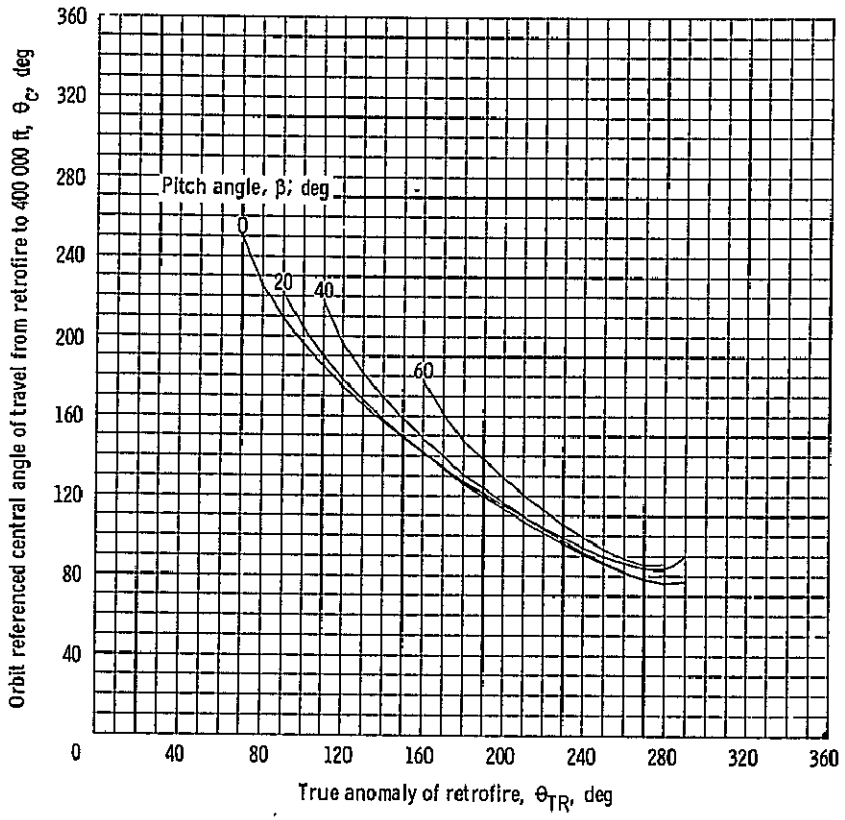
(c) Retrograde ΔV = 500 fps.

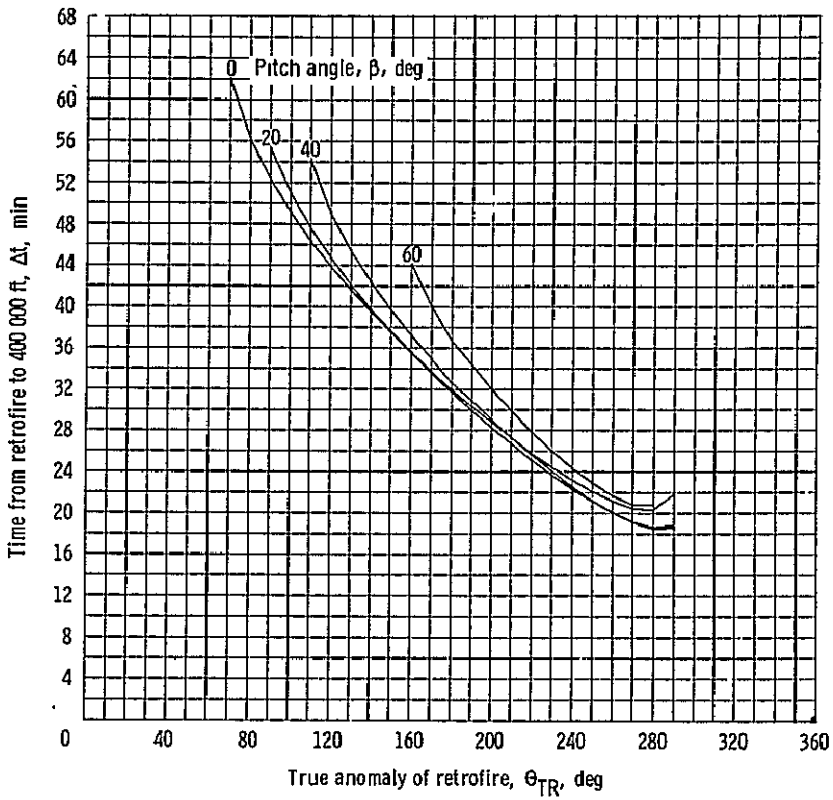
Figure 11. – Continued.



(d) Retrograde ΔV = 700 fps.

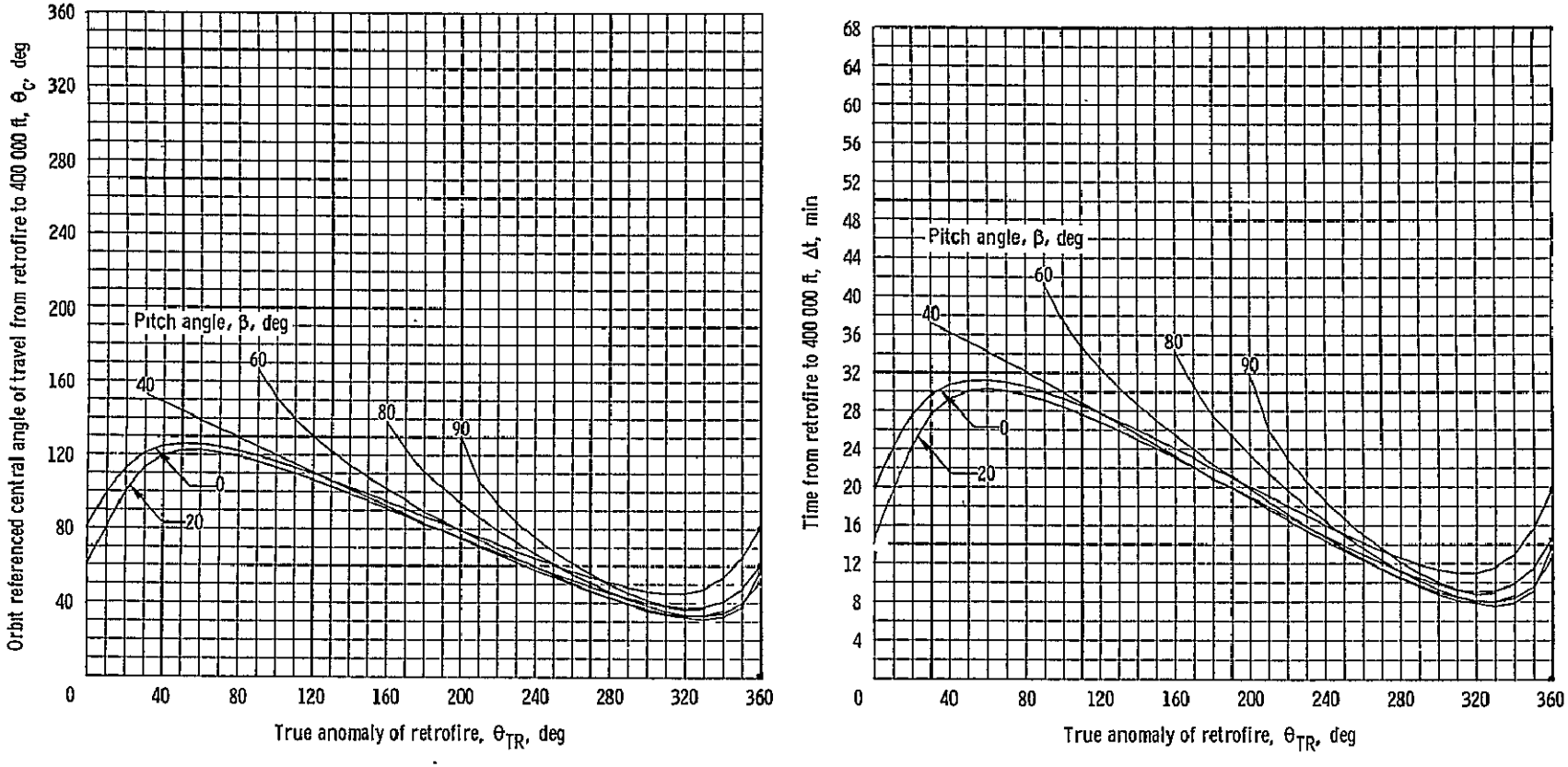
Figure 11.- Concluded.





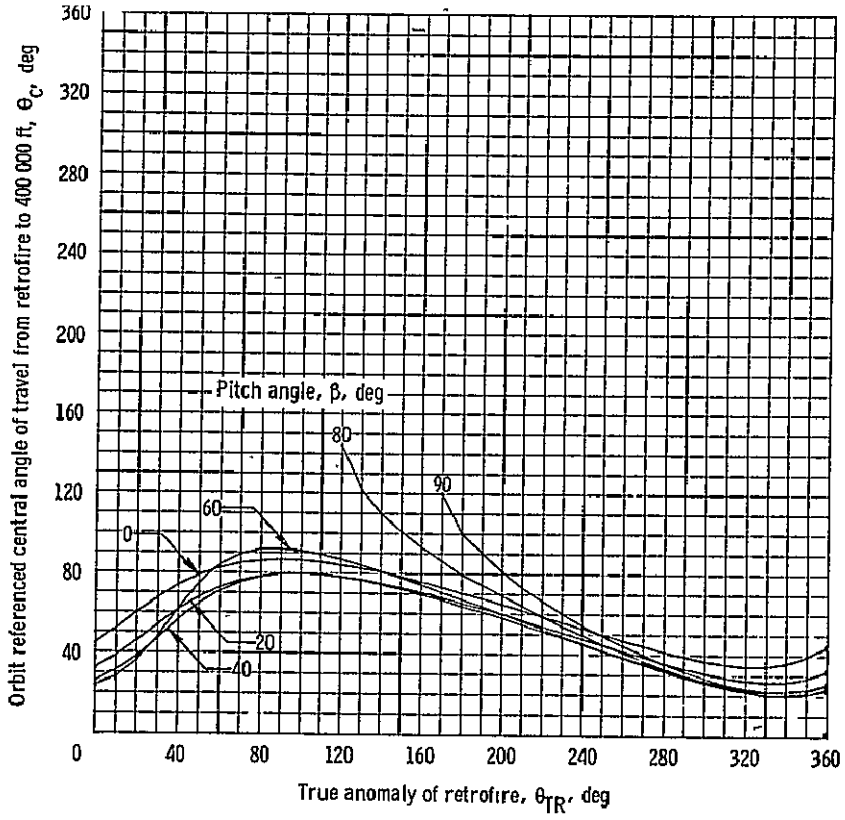
(a) Retrograde ΔV = 100 fps.

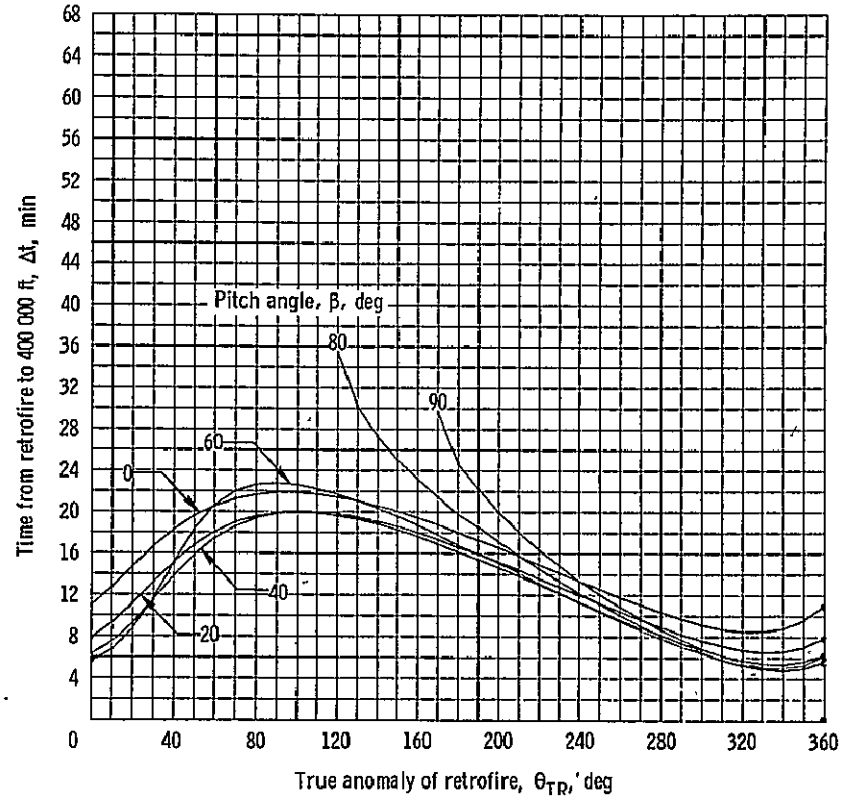
Figure 12. - Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 200 nautical miles and h_p = 90 nautical miles.



(b) Retrograde ΔV = 300 fps.

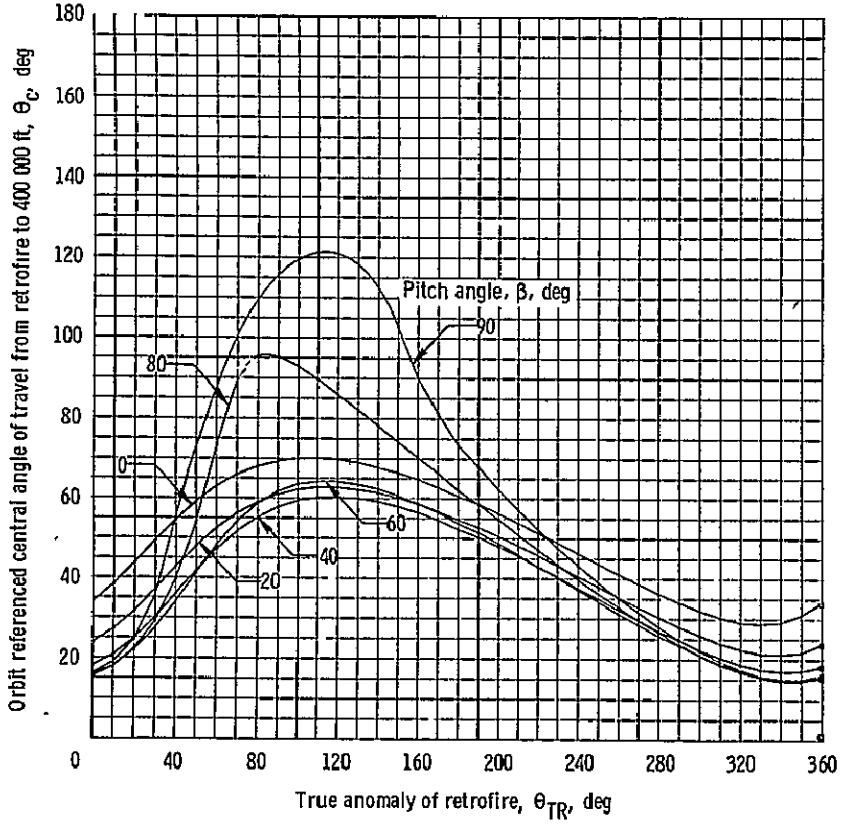
Figure 12. - Continued.

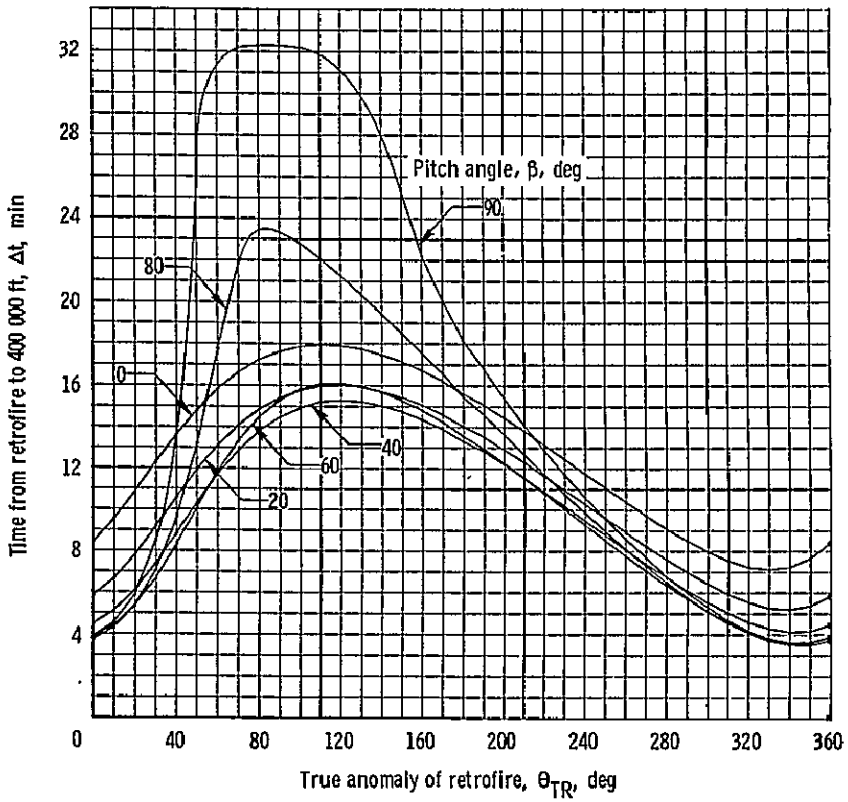




(c) Retrograde ΔV = 500 fps.

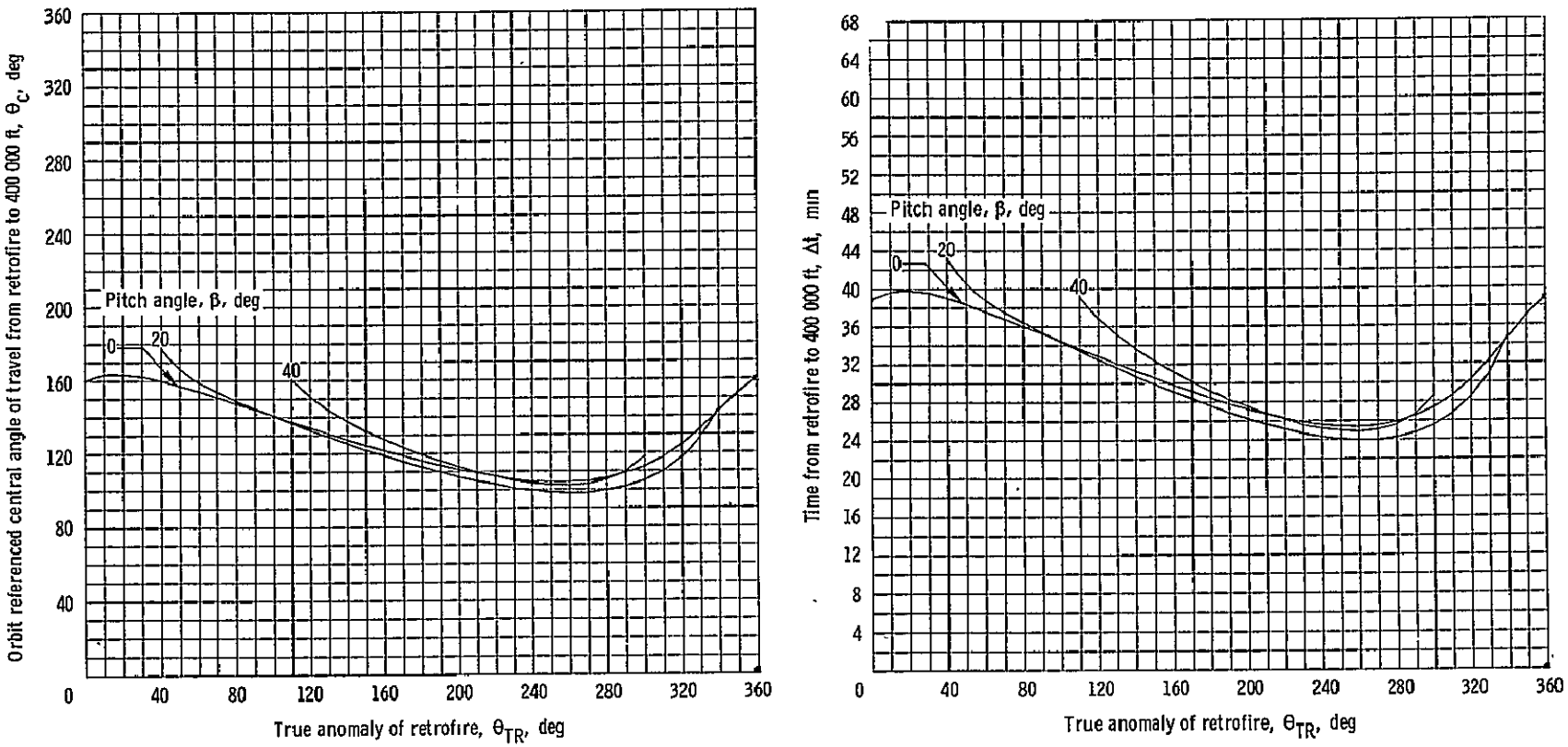
Figure 12.- Continued.





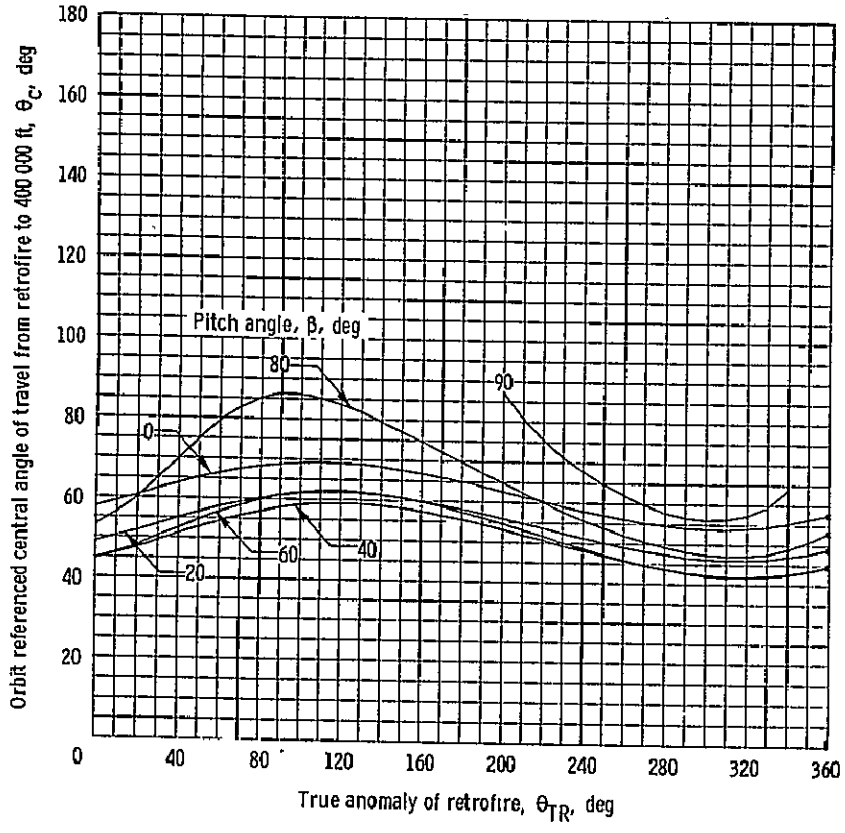
(d) Retrograde ΔV = 700 fps.

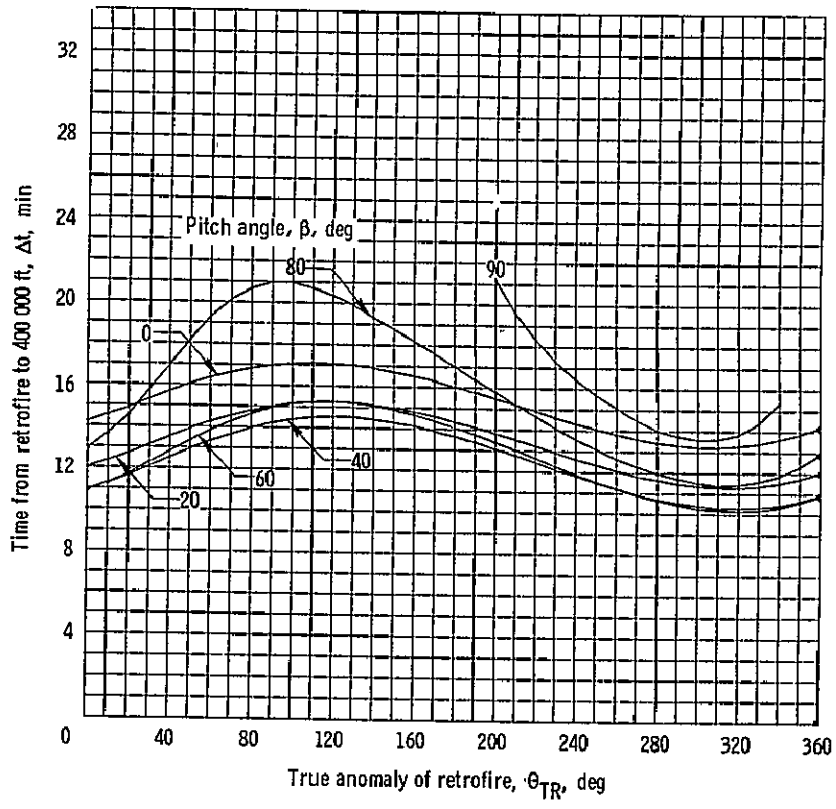
Figure 12.- Concluded.



(a) Retrograde ΔV = 100 fps.

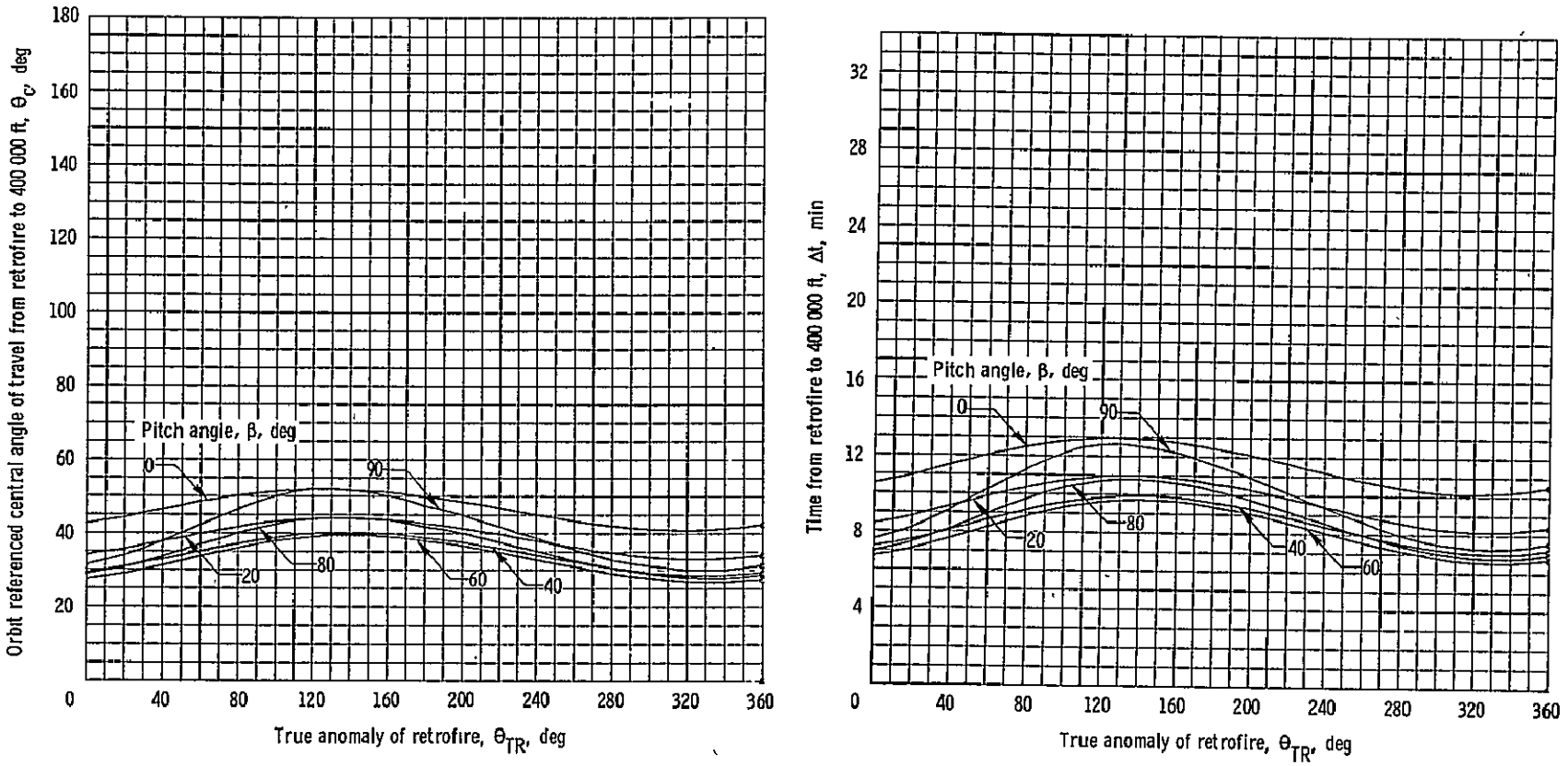
Figure 13. - Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 120 nautical miles and h_p = 100 nautical miles.





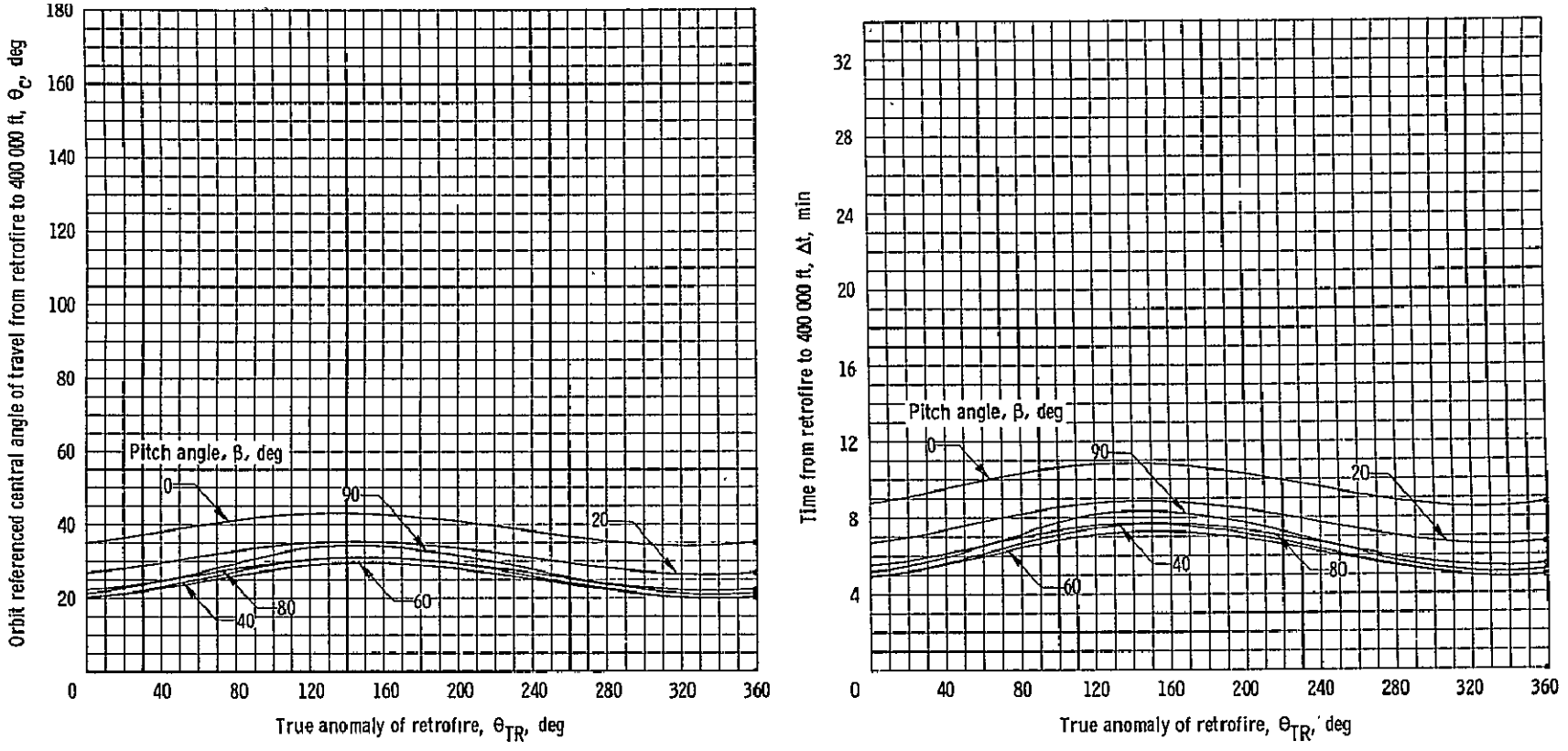
(b) Retrograde ΔV = 300 fps.

Figure 13.- Continued.



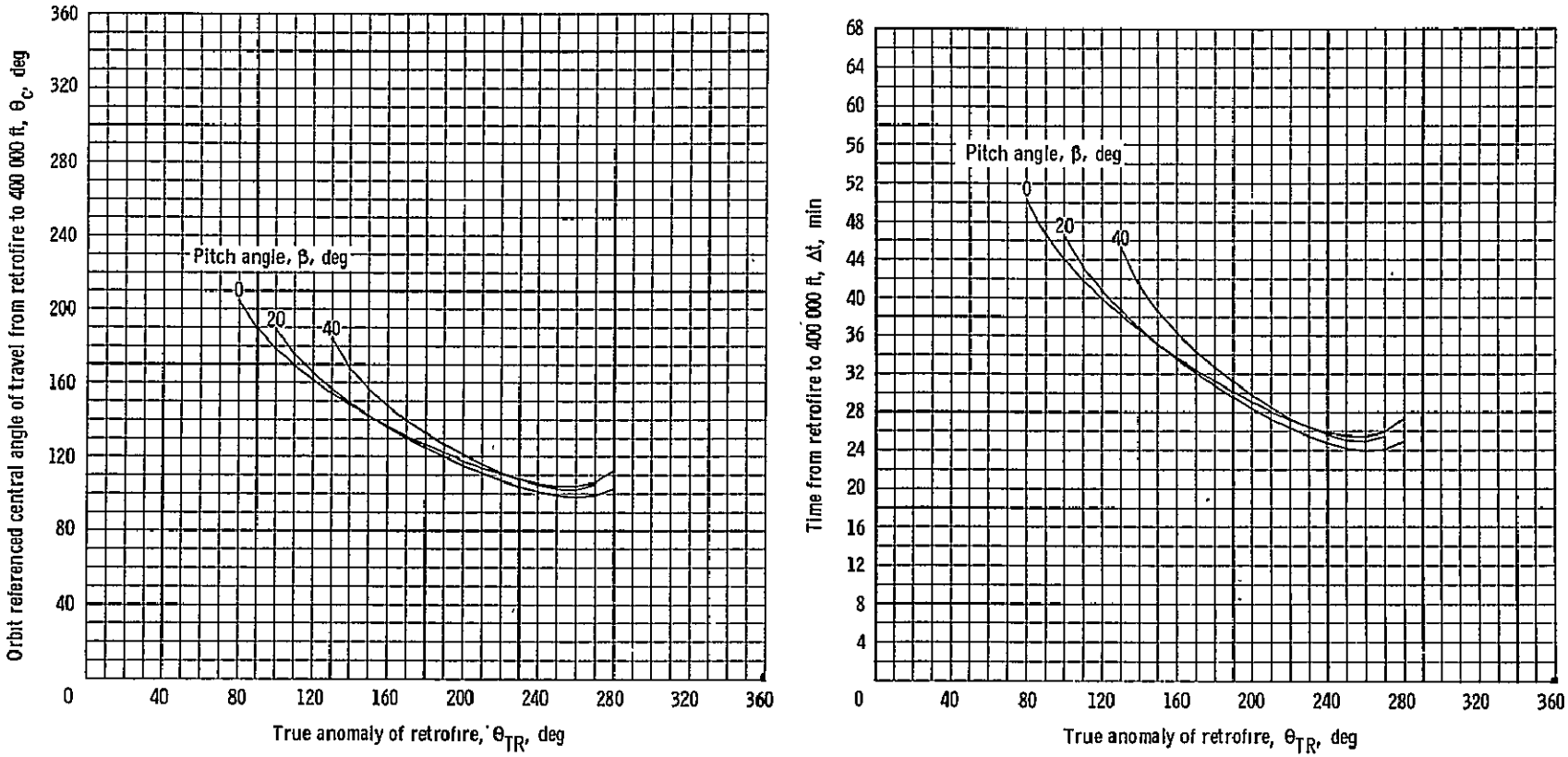
(c) Retrograde ΔV = 500 fps.

Figure 13.- Continued.



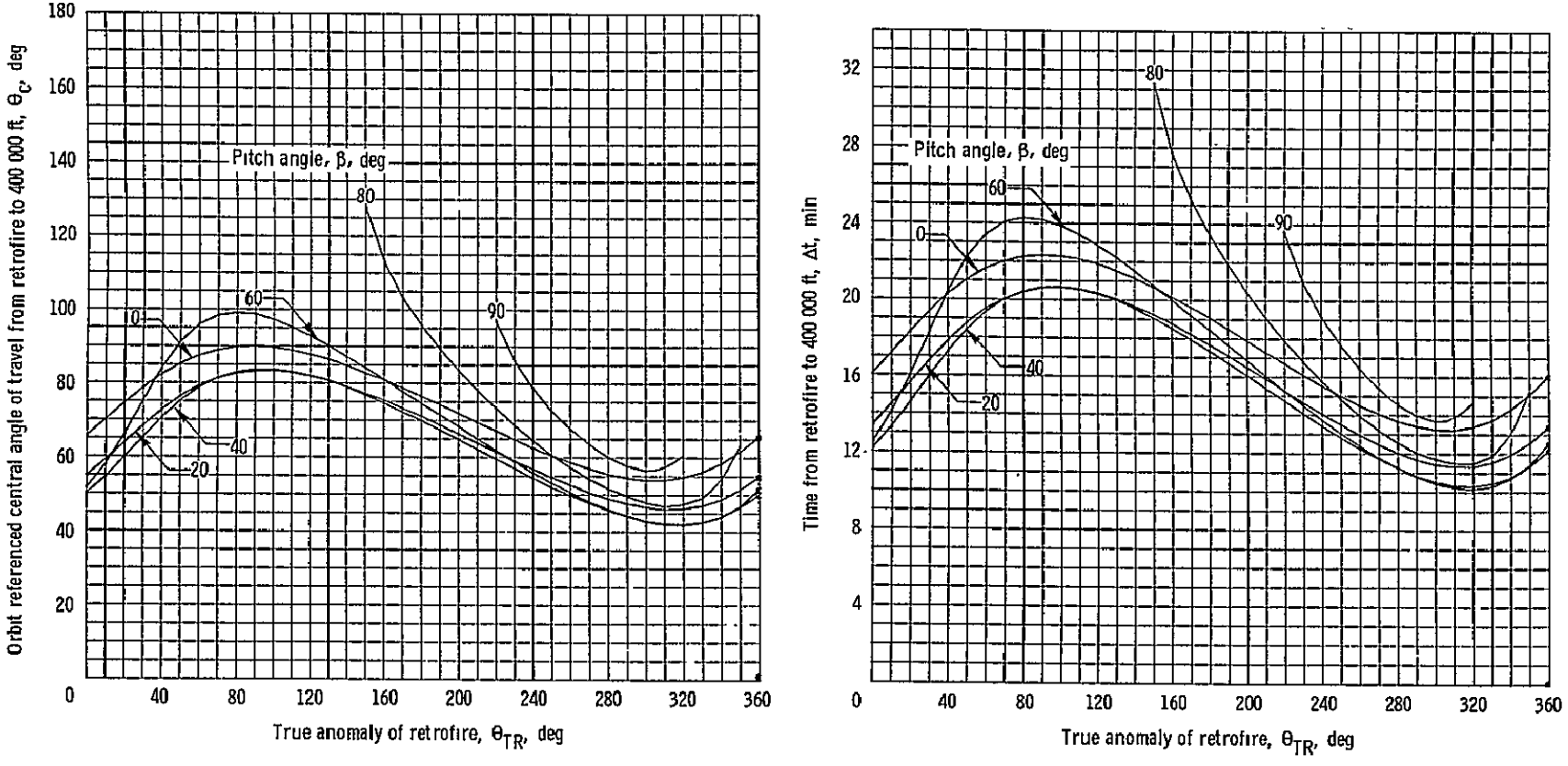
(d) Retrograde ΔV = 700 fps.

Figure 13.- Concluded.



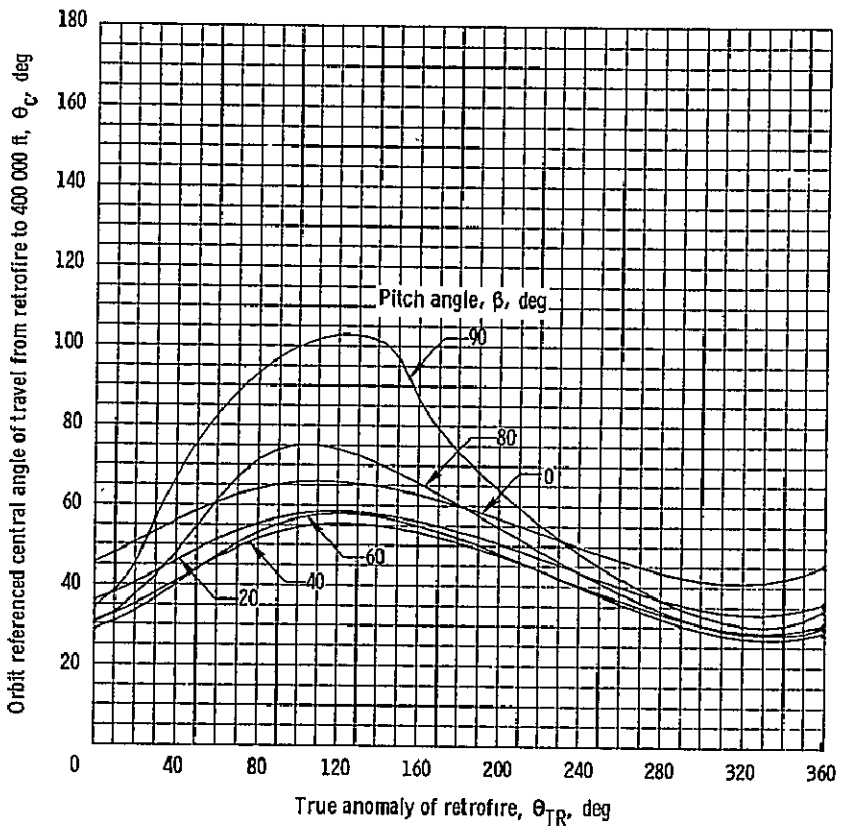
(a) Retrograde ΔV = 100 fps.

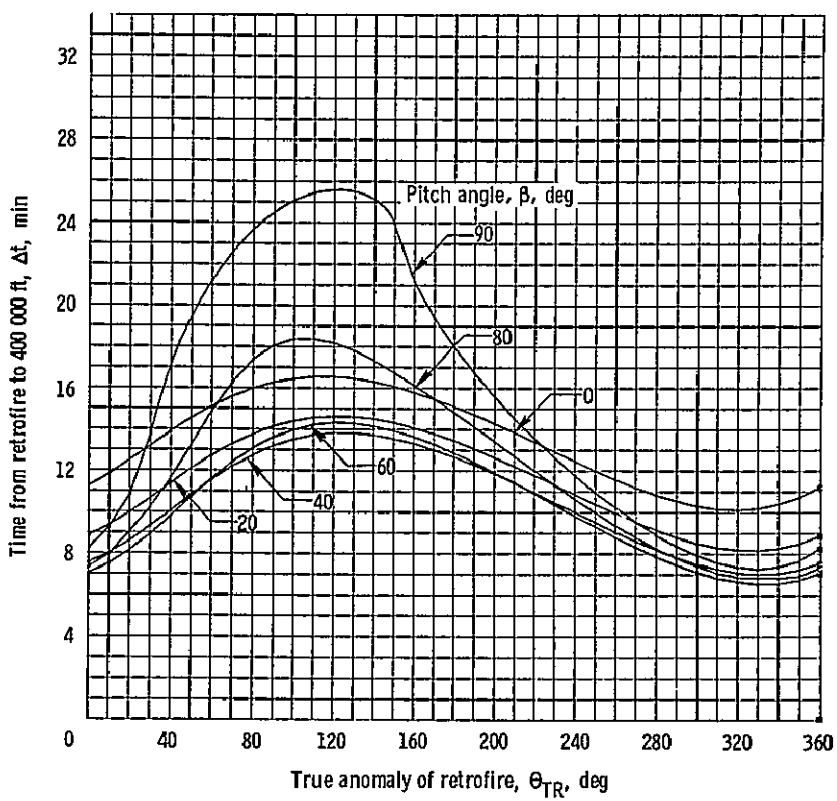
Figure 14.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 150 nautical miles and h_p = 100 nautical miles.



(b) Retrograde ΔV = 300 fps.

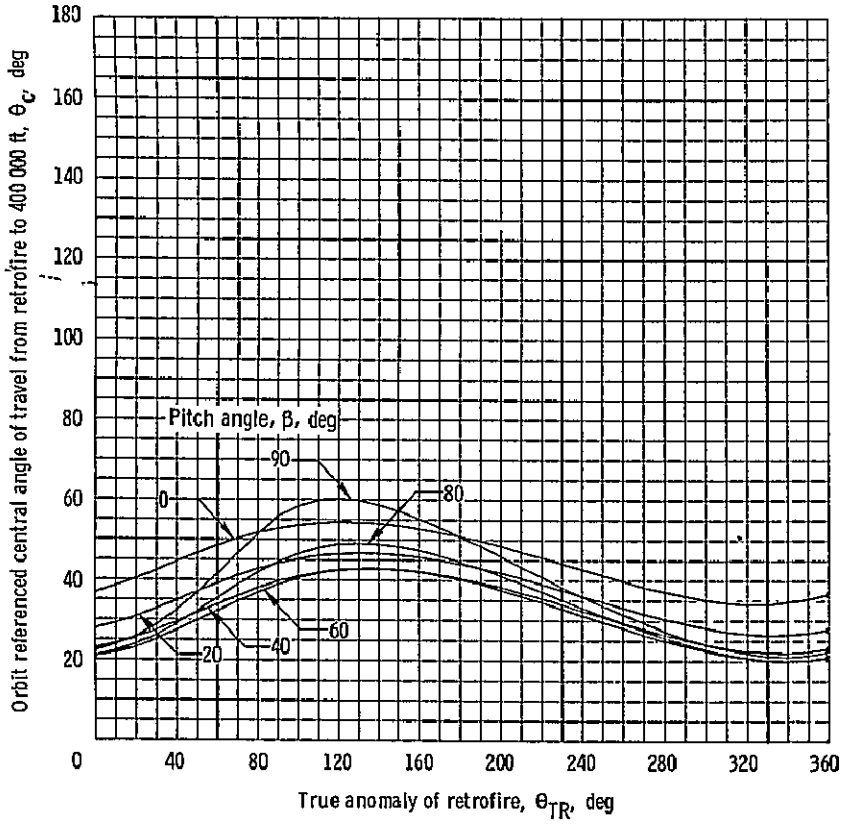
Figure 14.- Continued.

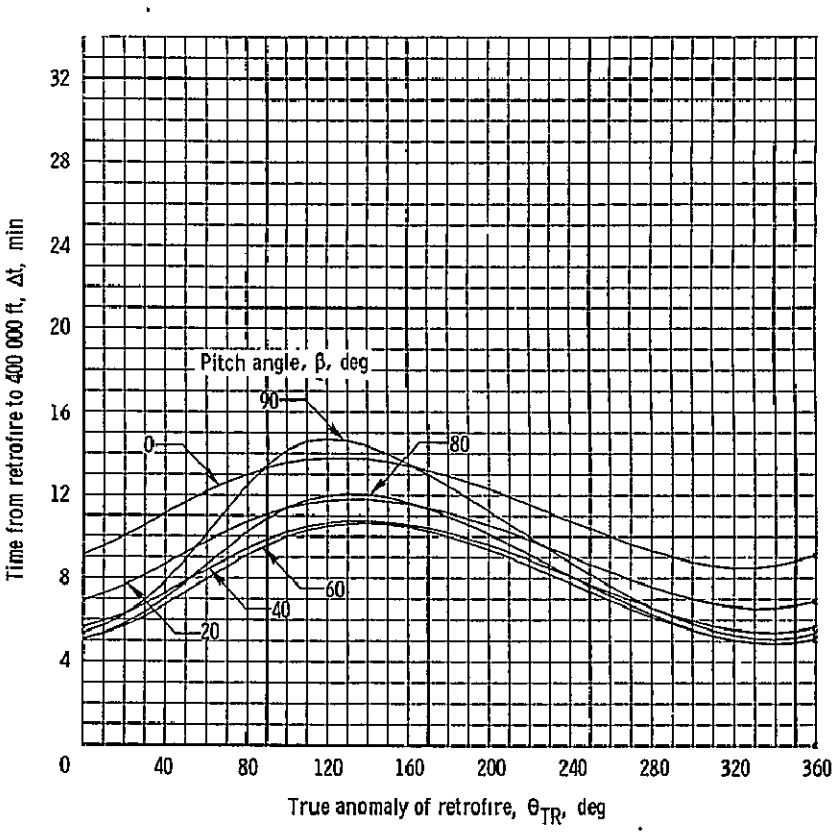




(c) Retrograde ΔV = 500 fps.

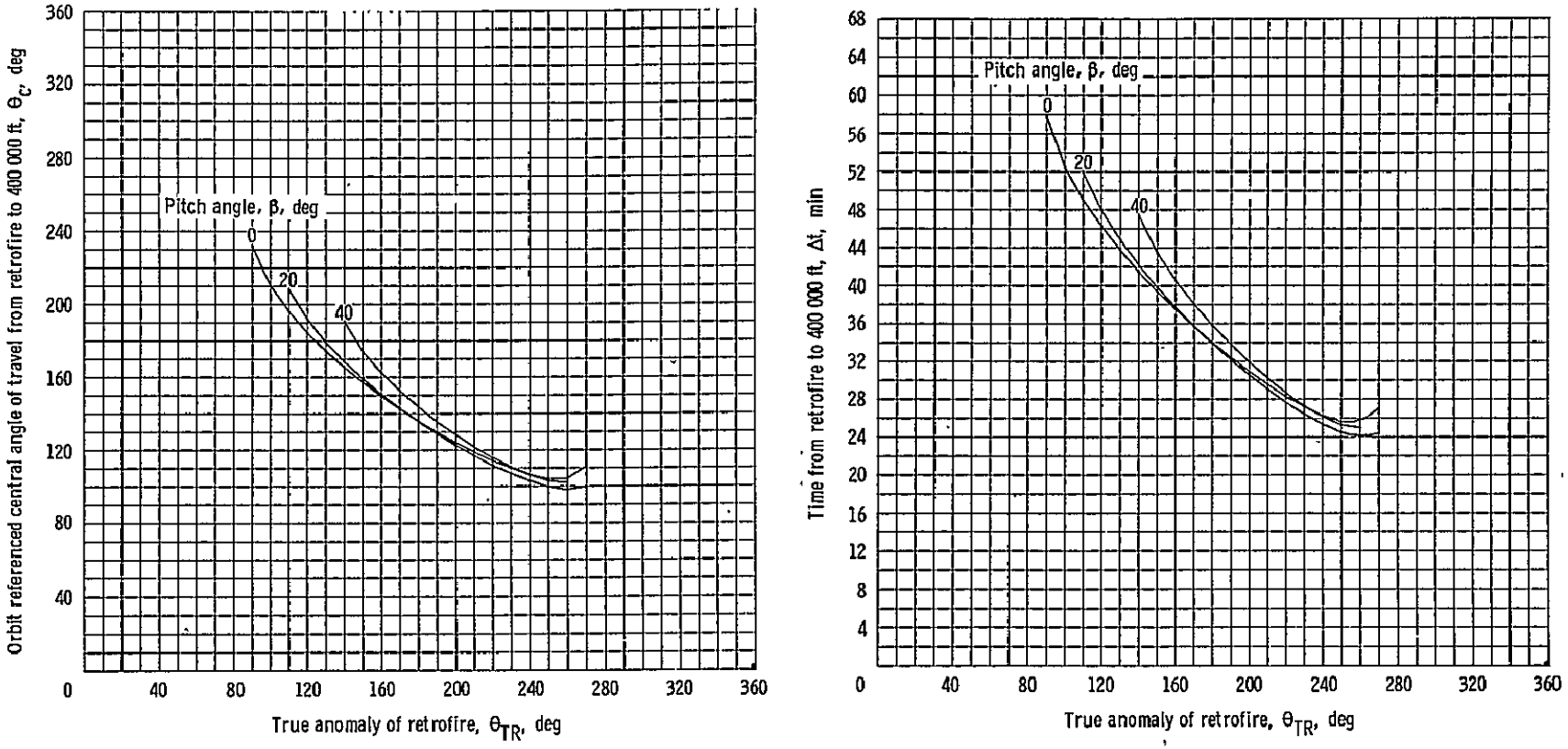
Figure 14.- Continued.





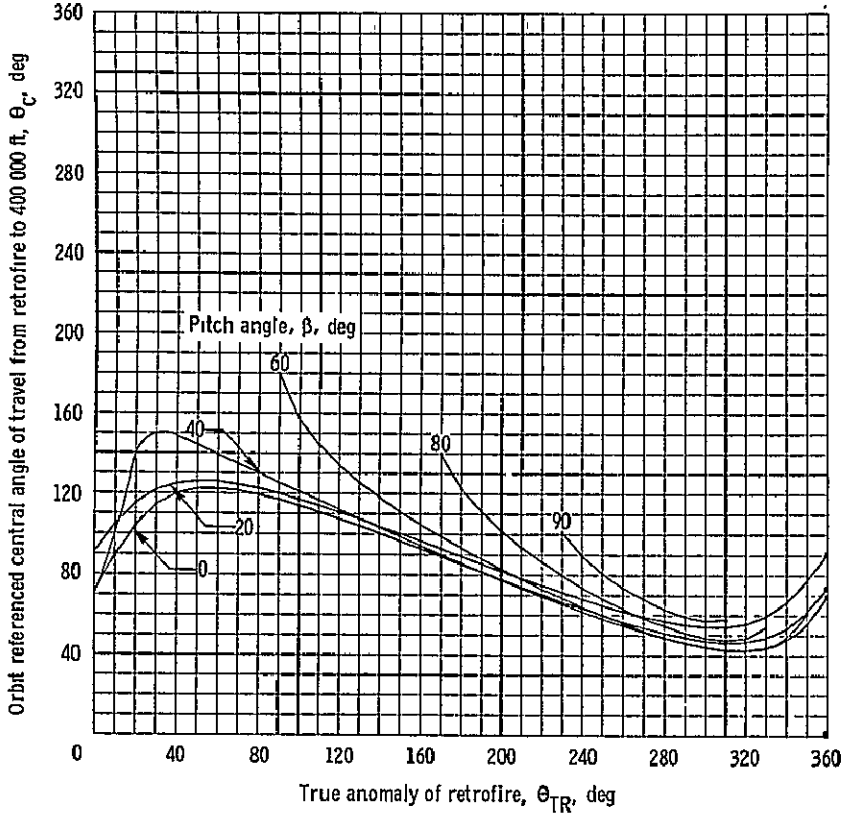
(d) Retrograde ΔV = 700 fps.

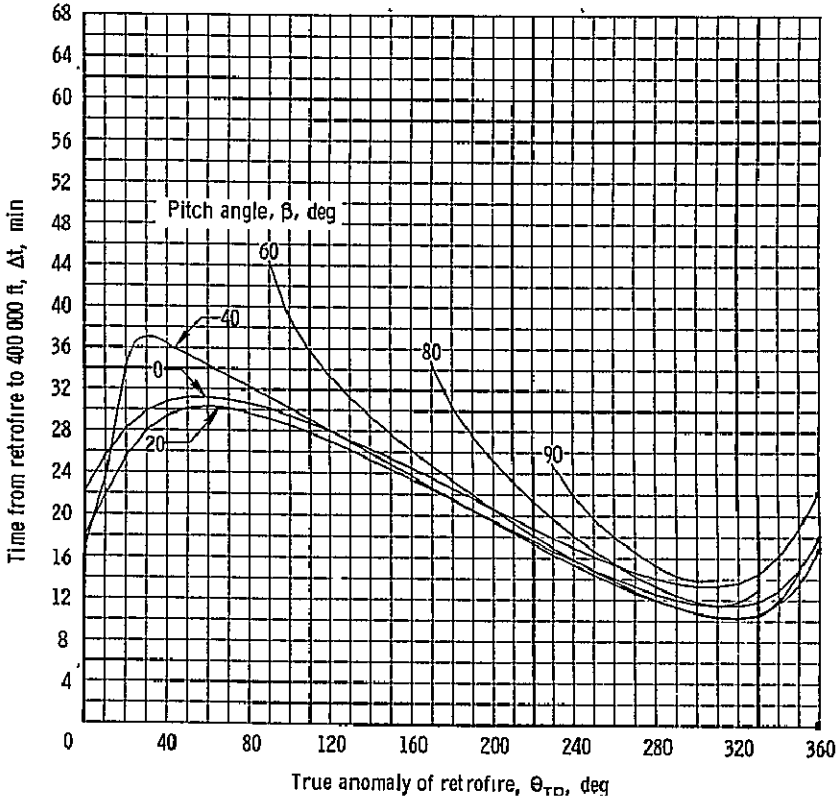
Figure 14.- Concluded.



(a) Retrograde ΔV = 100 fps.

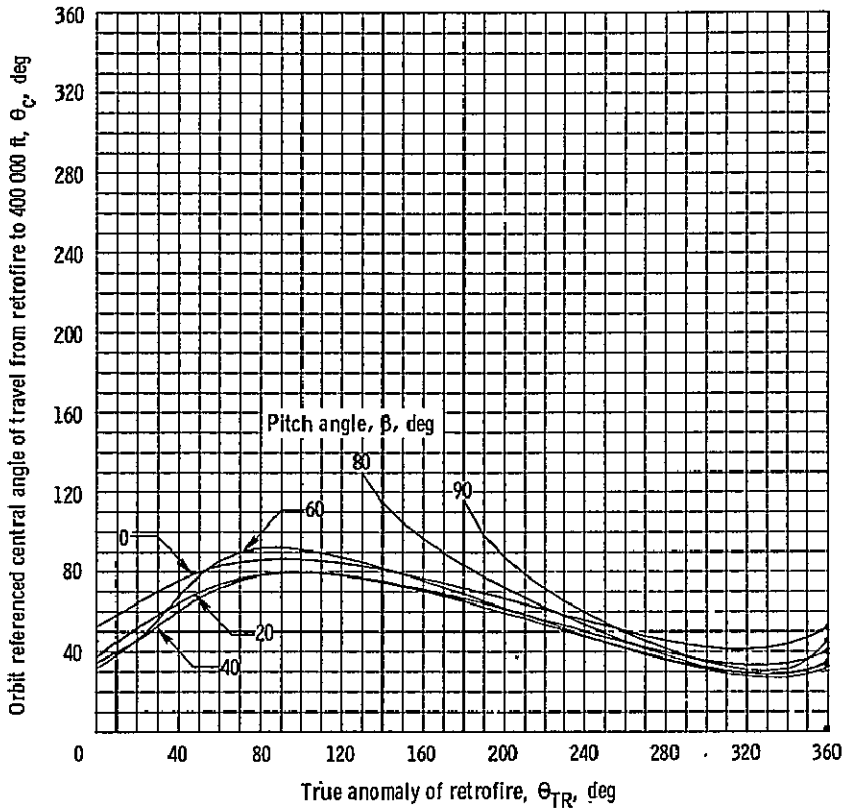
Figure 15. - Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 200 nautical miles and h_p = 100 nautical miles.

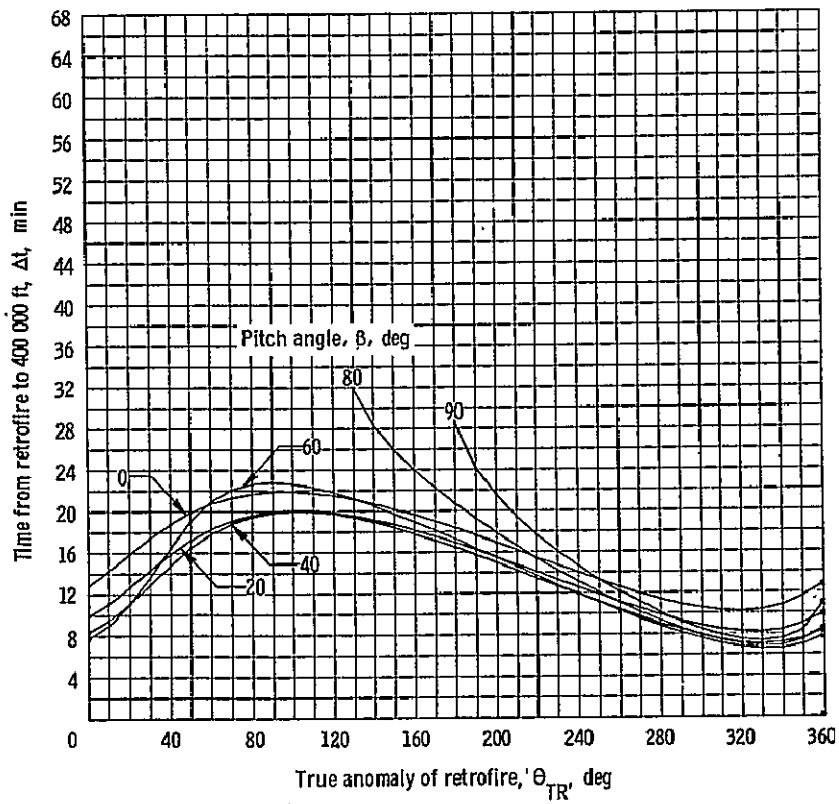




(b) Retrograde ΔV = 300 fps.

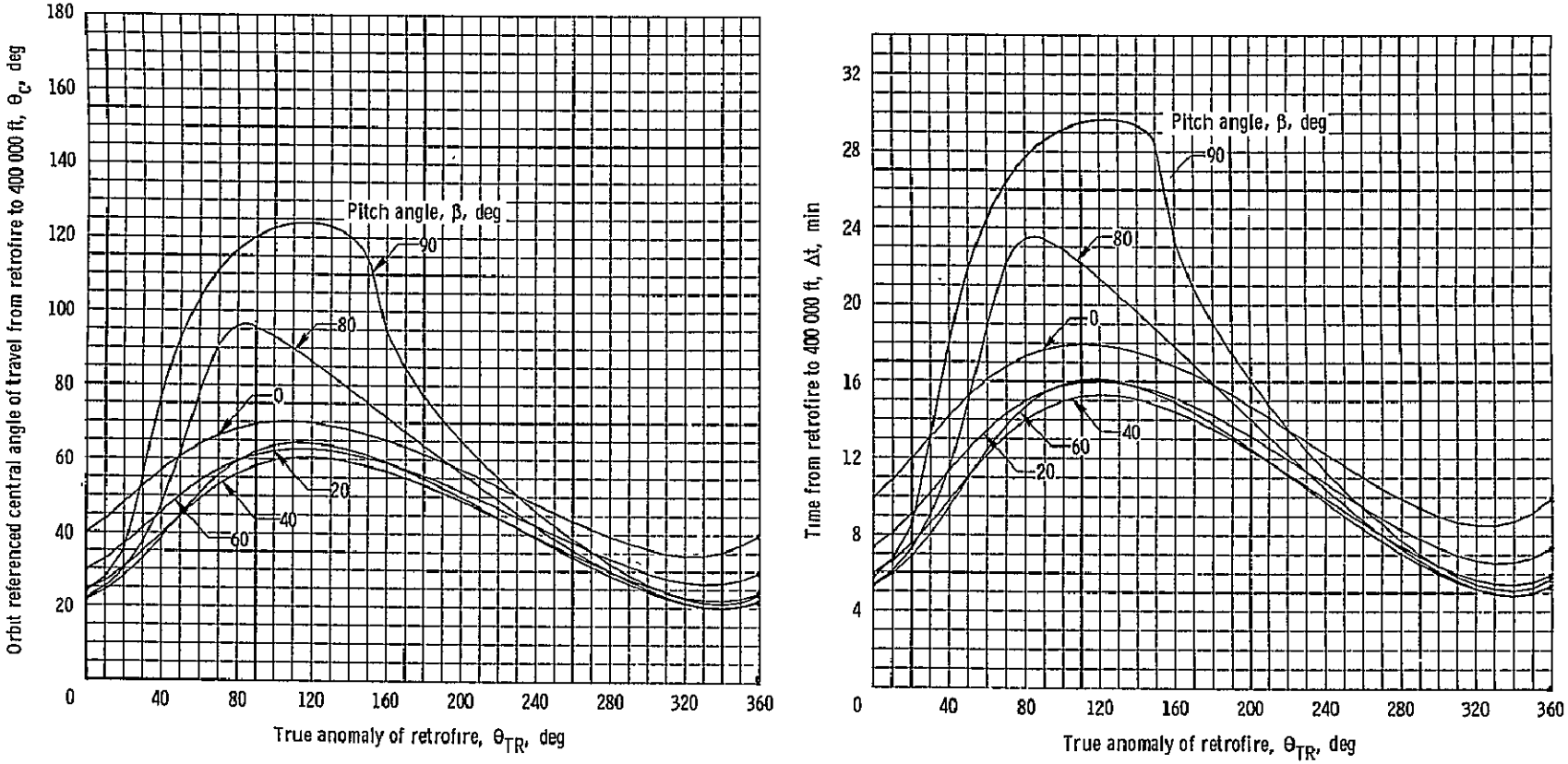
Figure 15. – Continued.





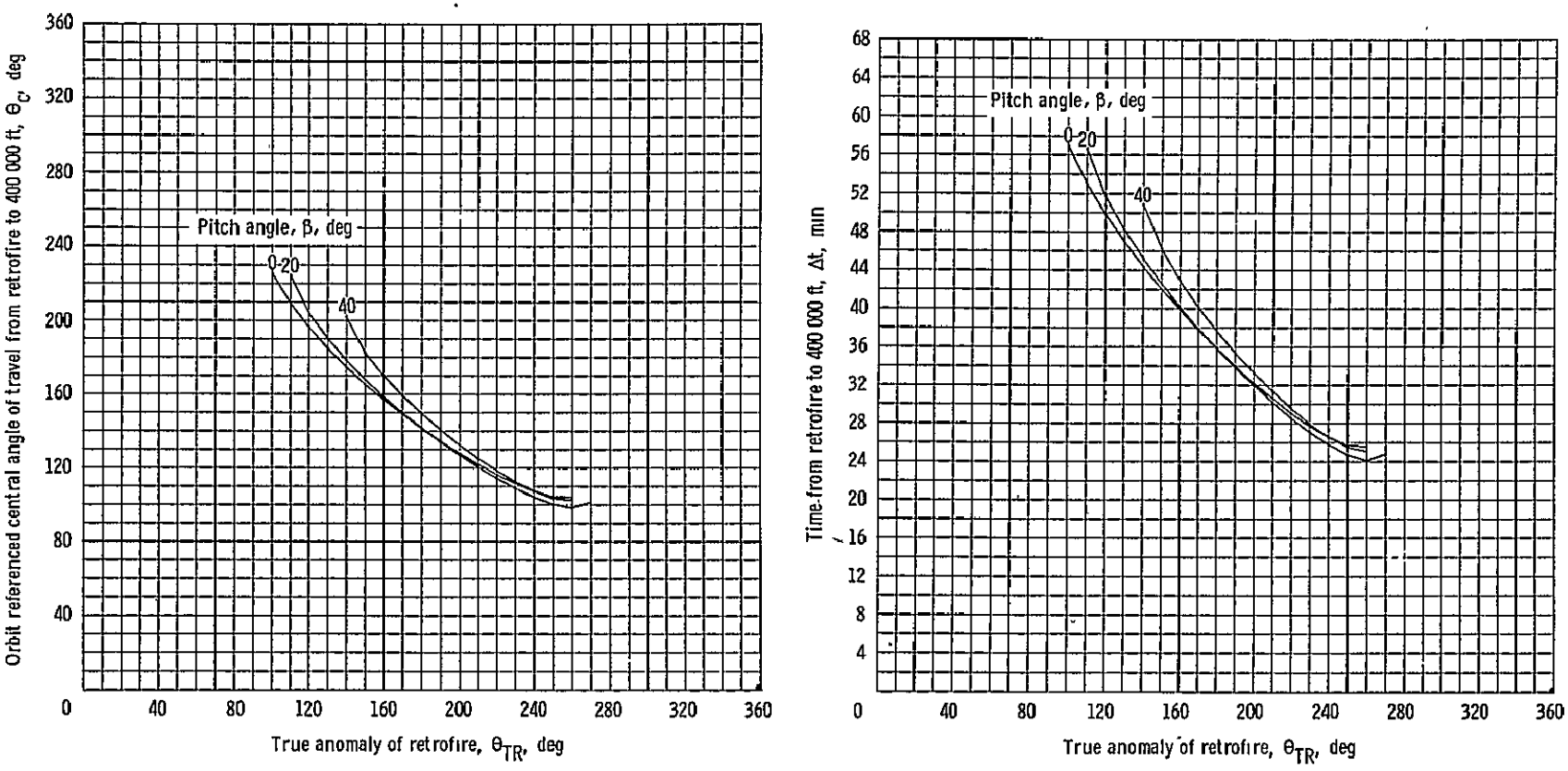
(c) Retrograde ΔV = 500 fps.

Figure 15. - Continued.



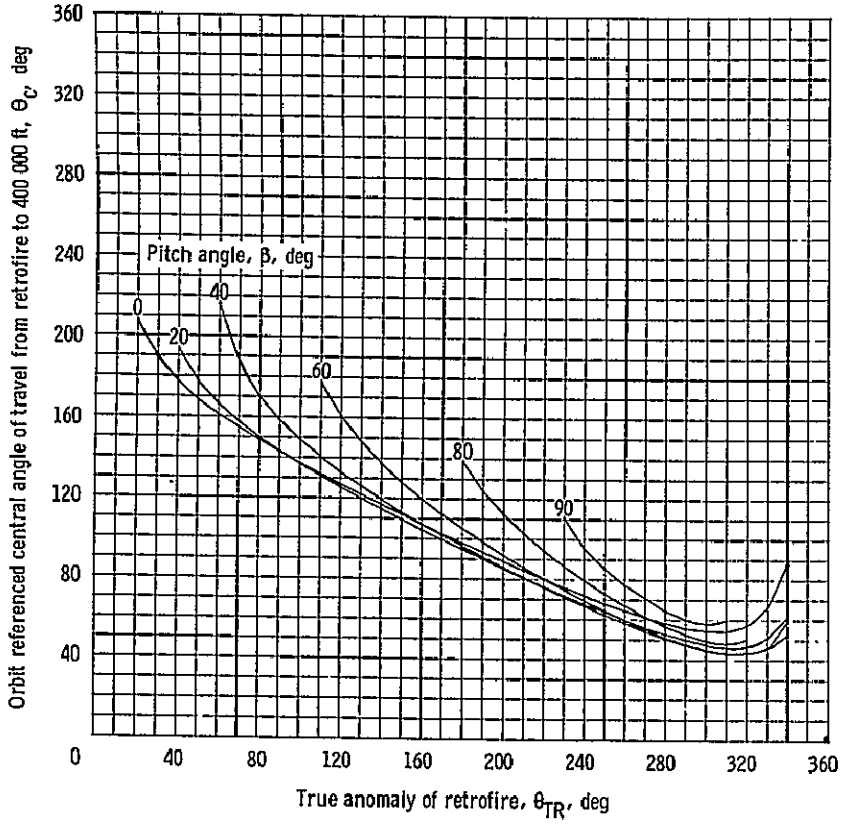
(d) Retrograde ΔV = 700 fps.

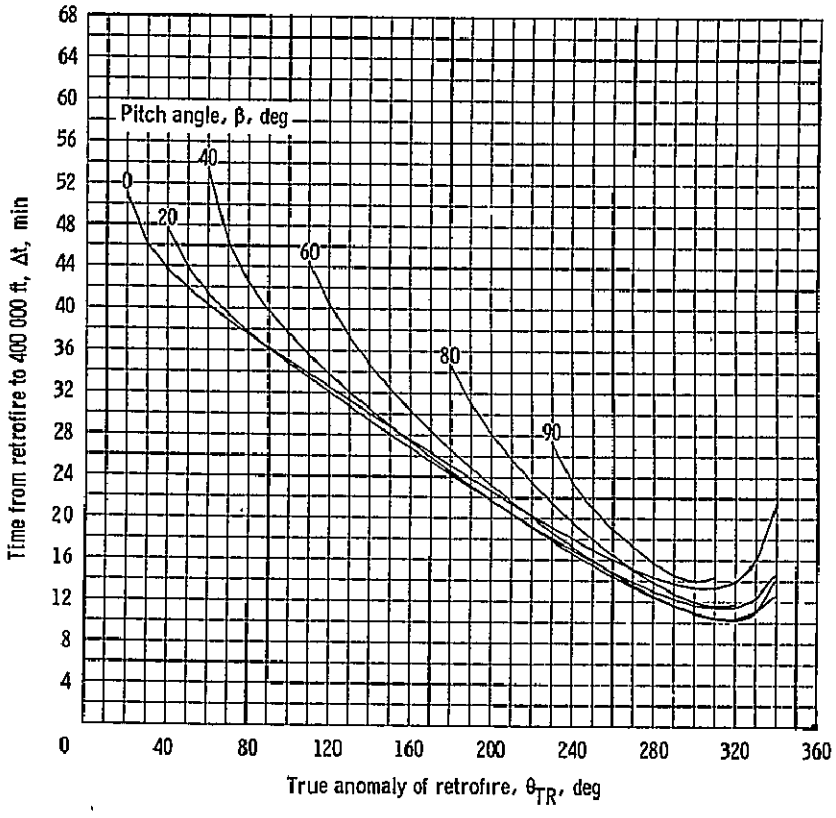
Figure 15. - Concluded.



(a) Retrograde ΔV = 100 fps.

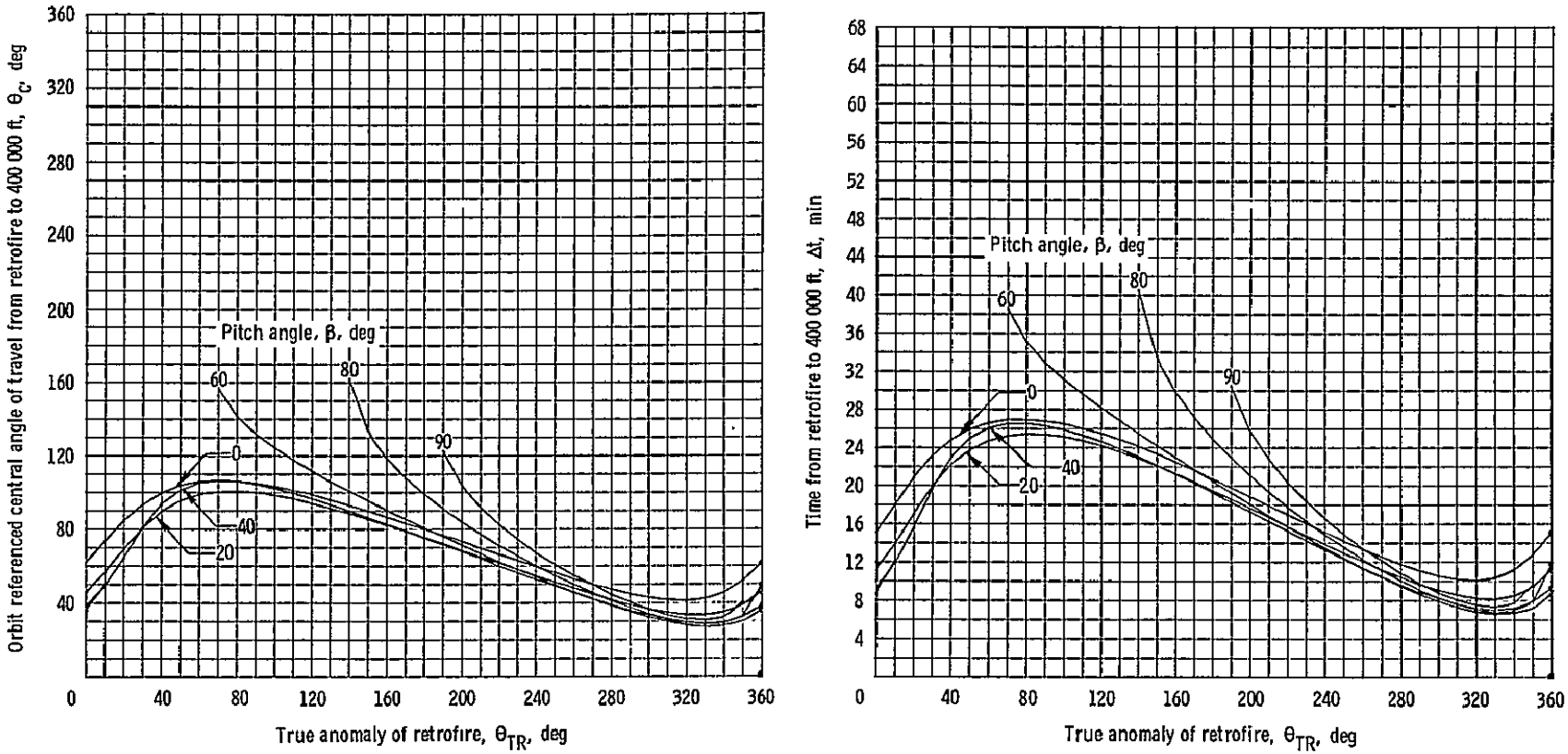
Figure 16.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 250 nautical miles and h_p = 100 nautical miles.





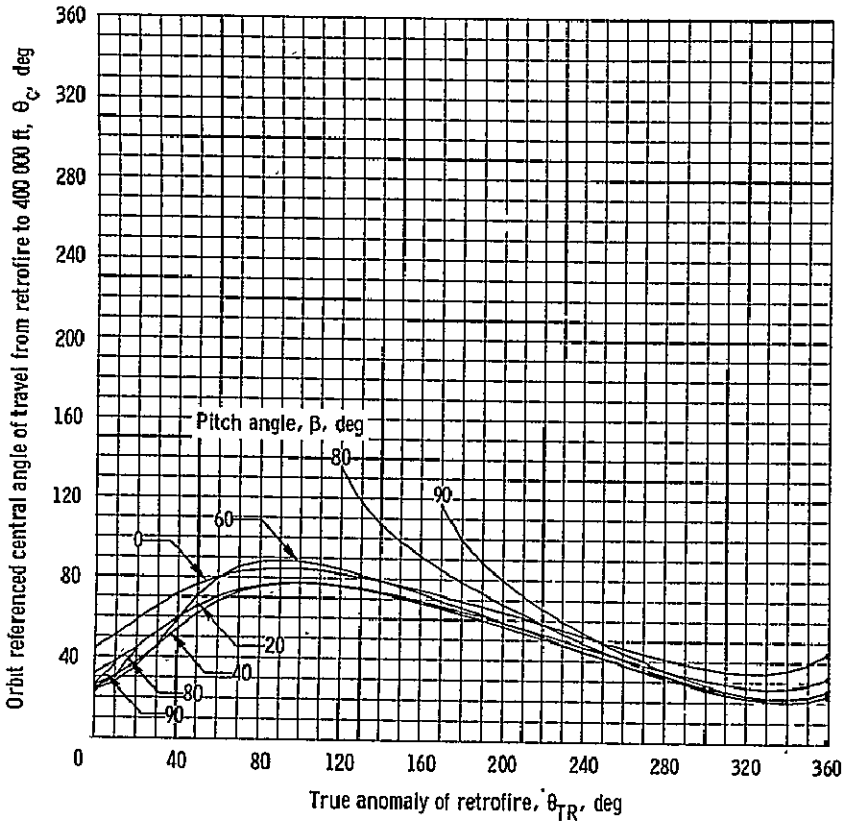
(b) Retrograde ΔV = 300 fps.

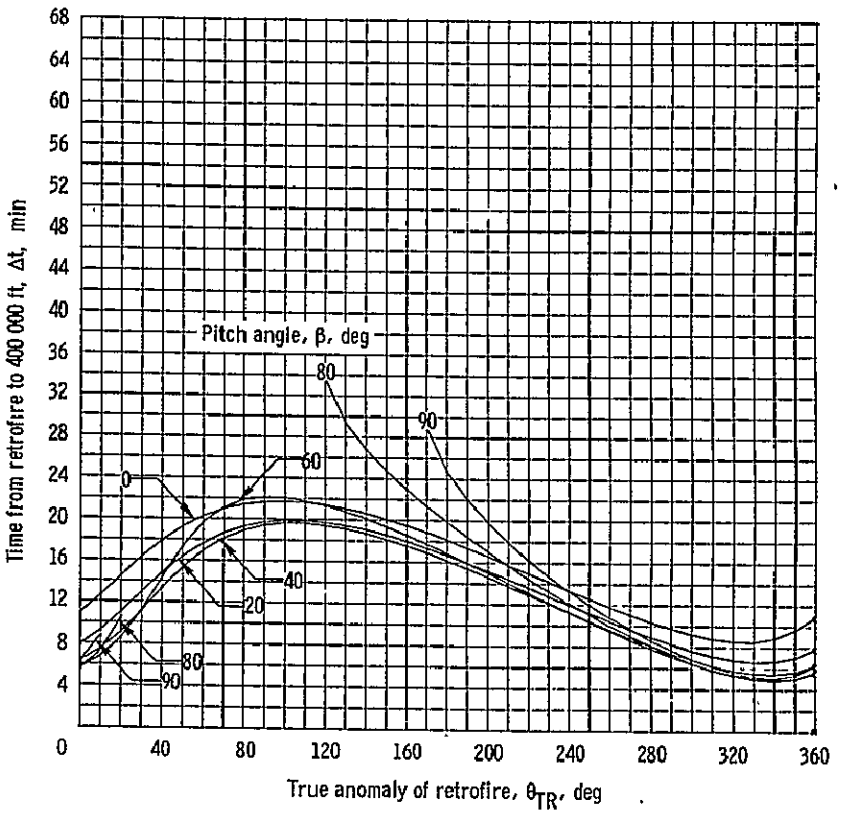
Figure 16. - Continued.



(c) Retrograde ΔV = 500 fps.

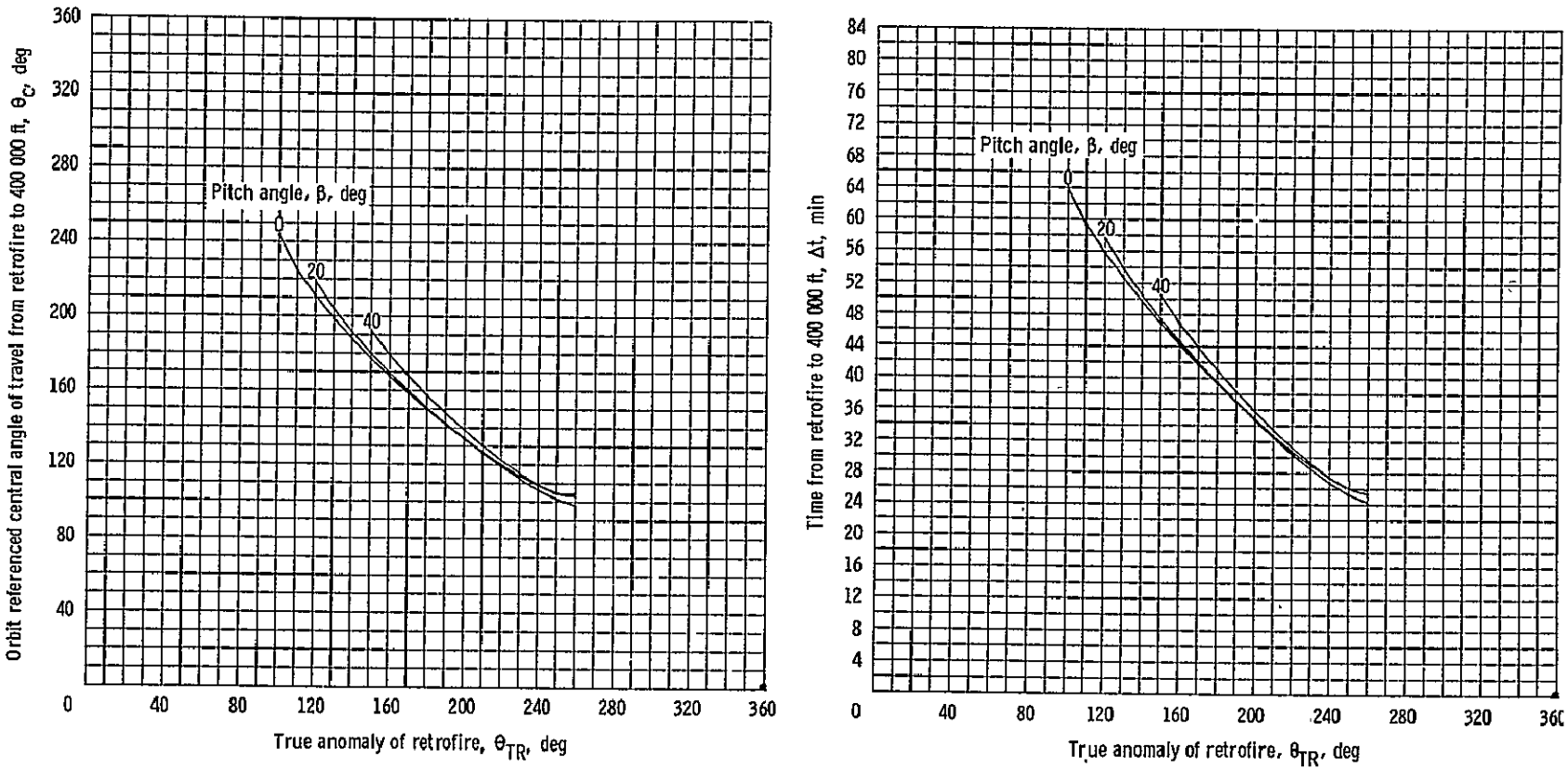
Figure 16. - Continued.





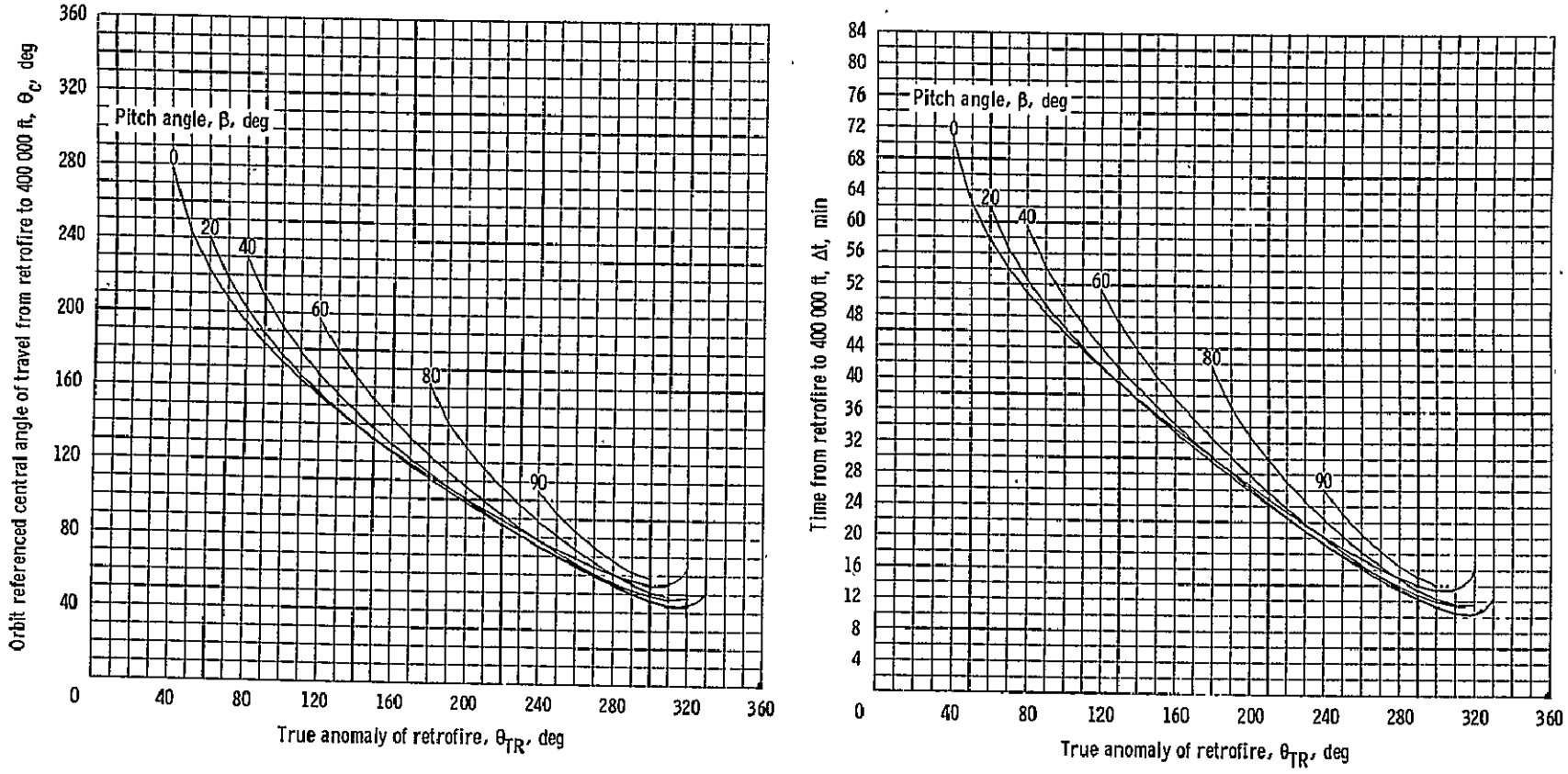
(d) Retrograde ΔV = 700 fps.

Figure 16. - Concluded.



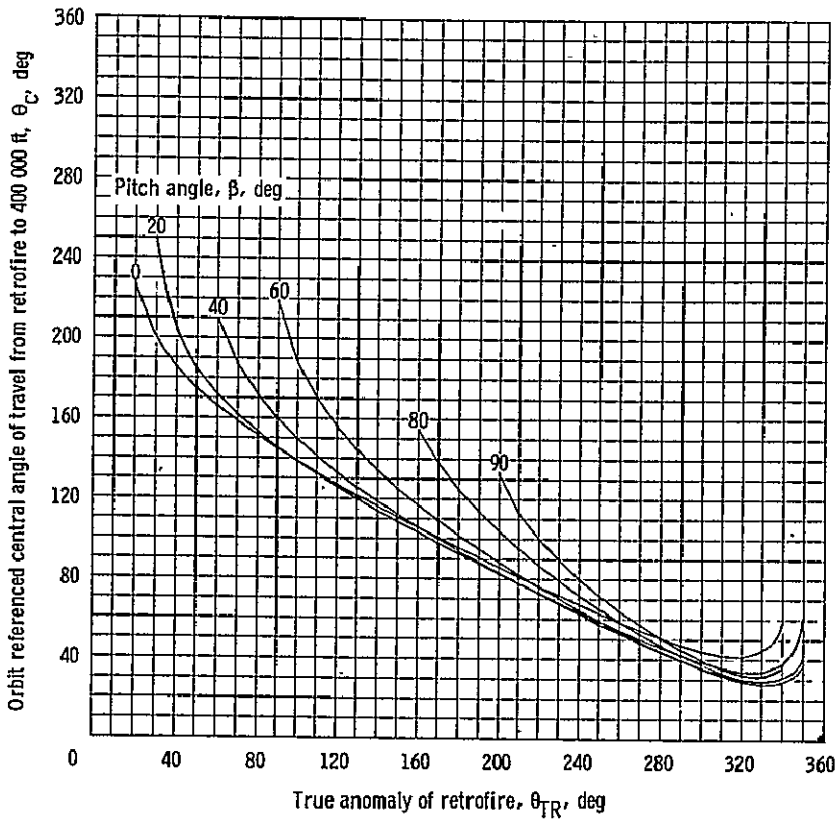
(a) Retrograde ΔV = 100 fps.

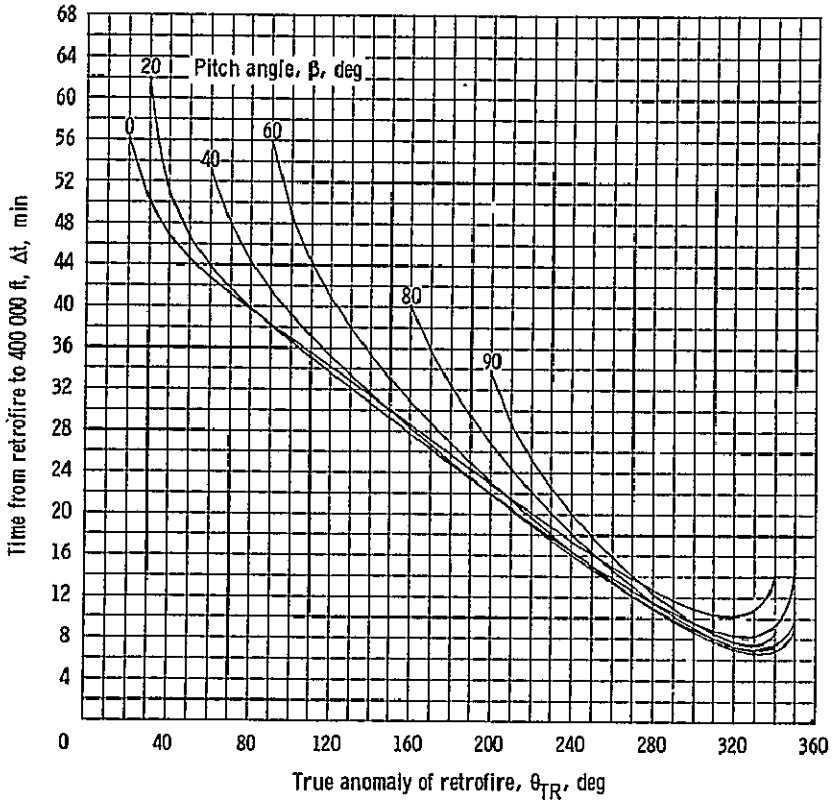
Figure 17.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 400 nautical miles and h_p = 100 nautical miles.



(b) Retrograde ΔV = 300 fps.

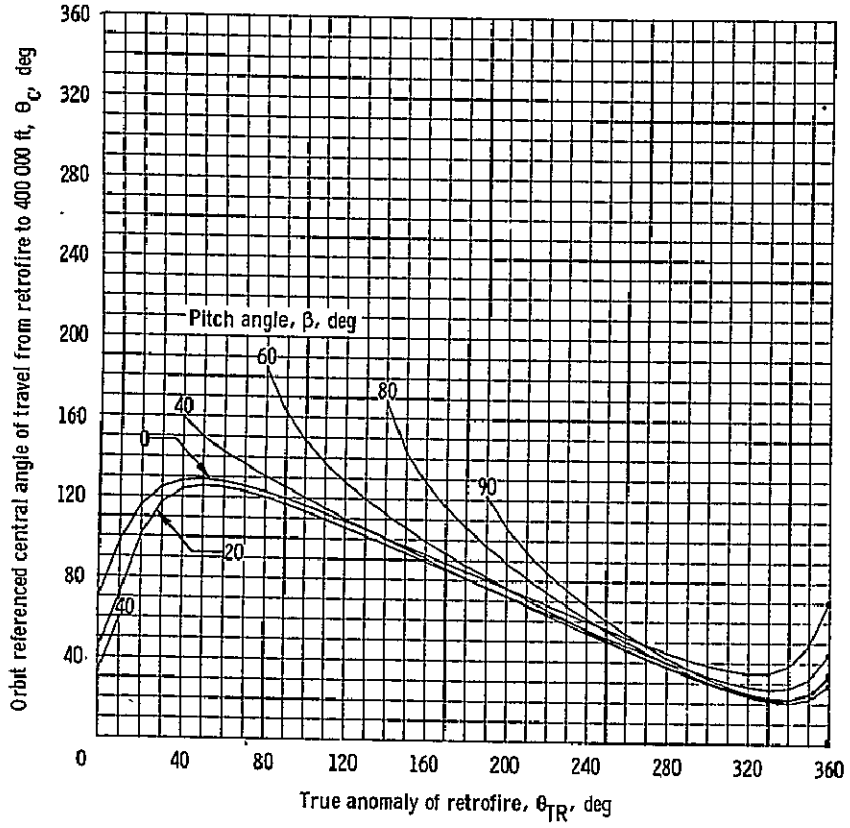
Figure 17.- Continued.

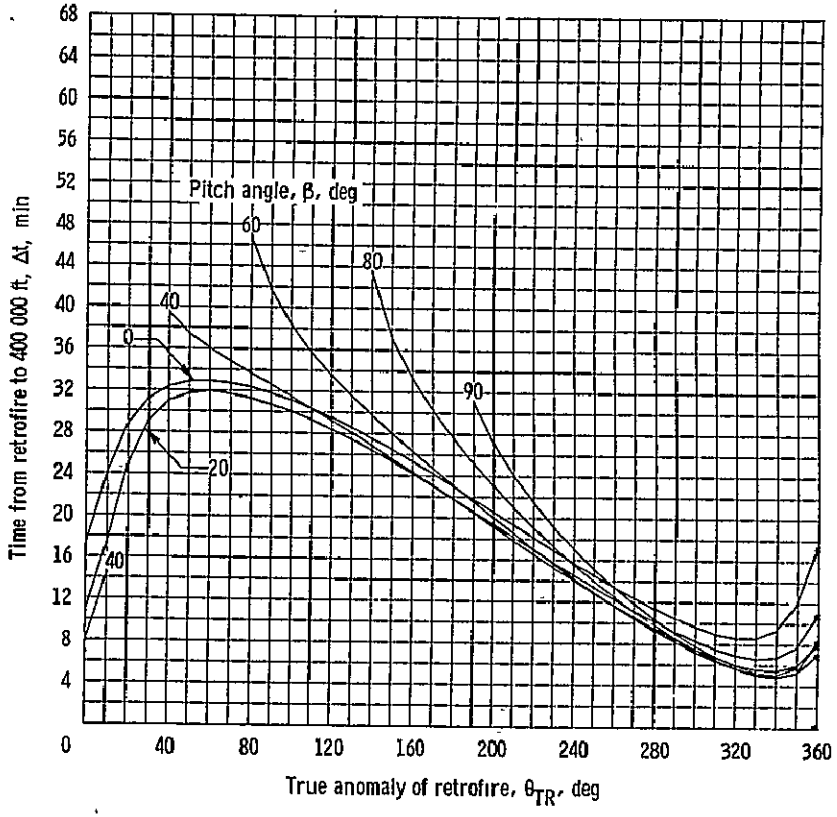




(c) Retrograde ΔV = 500 fps.

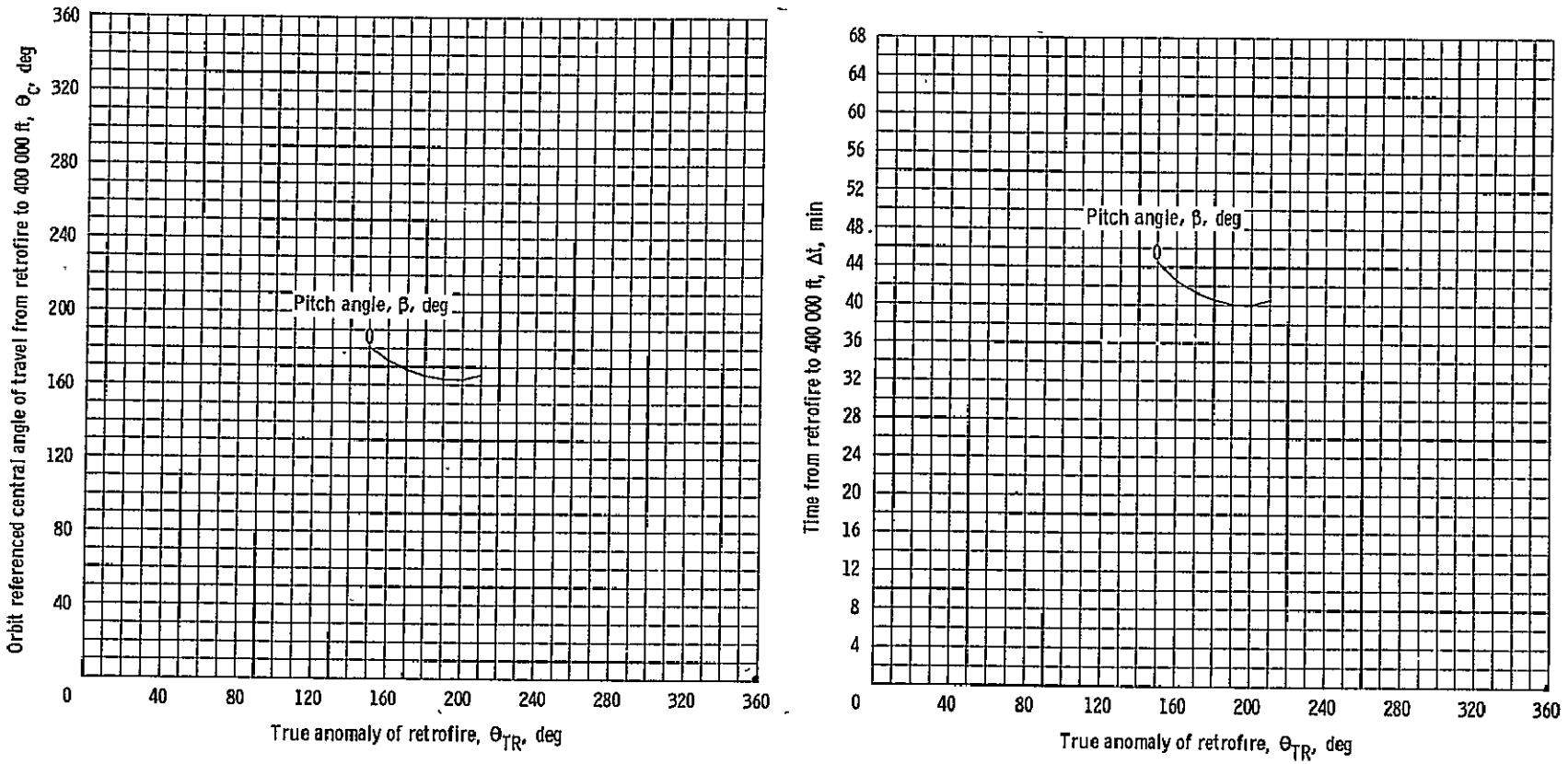
Figure 17. - Continued.





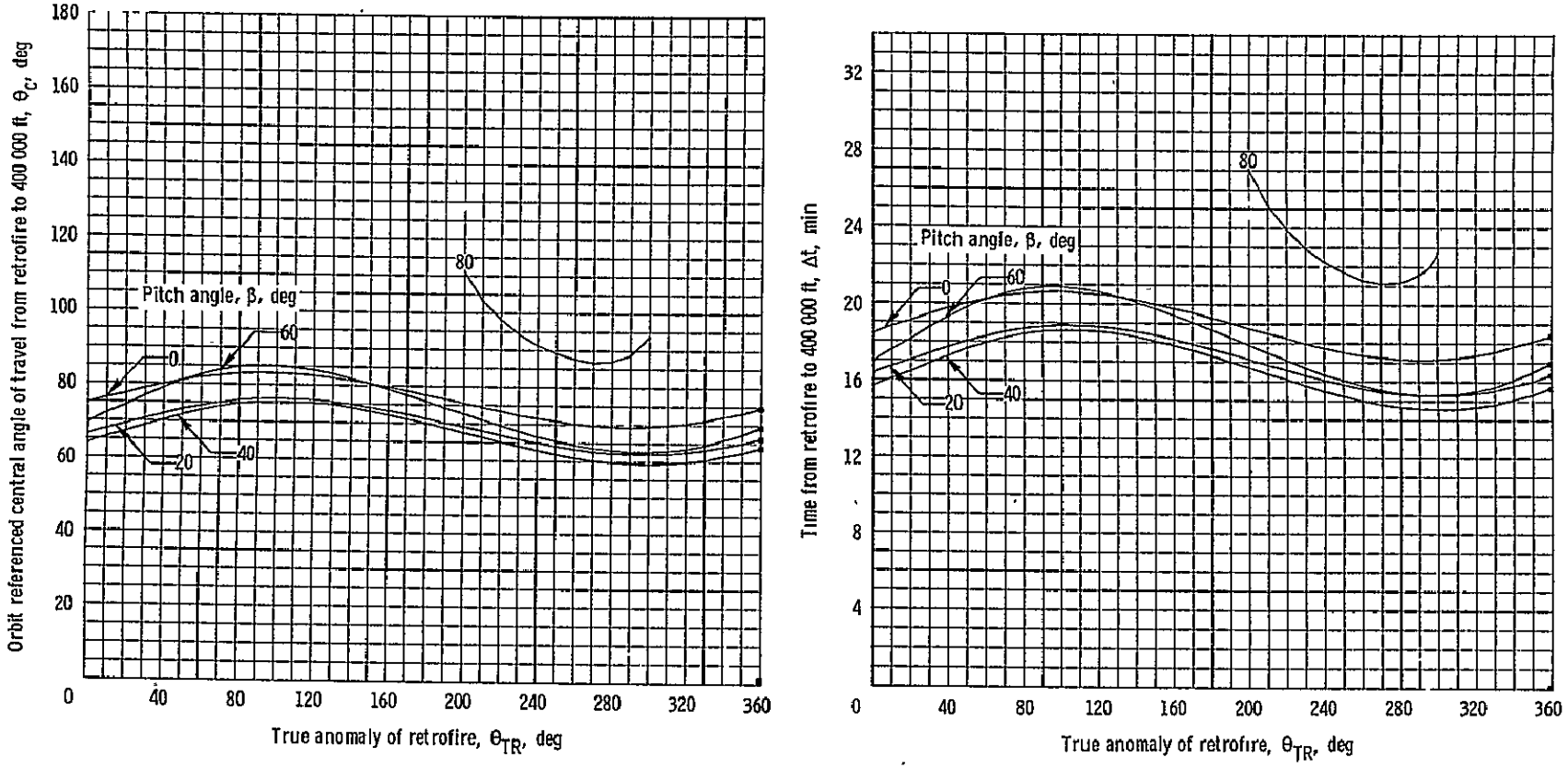
(d) Retrograde ΔV = 700 fps.

Figure 17.- Concluded.



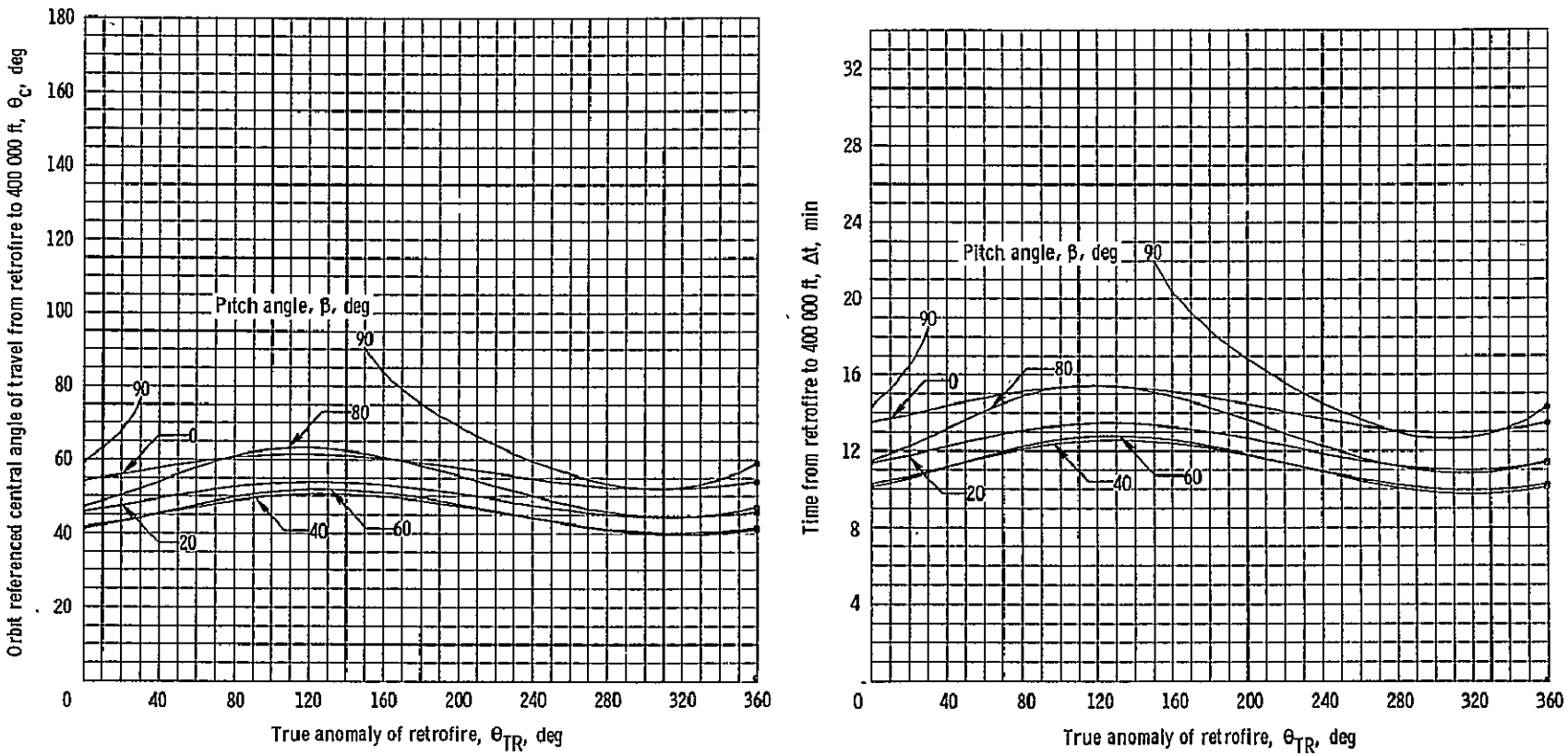
(a) Retrograde ΔV = 100 fps.

Figure 18.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 140 nautical miles and h_p = 120 nautical miles.



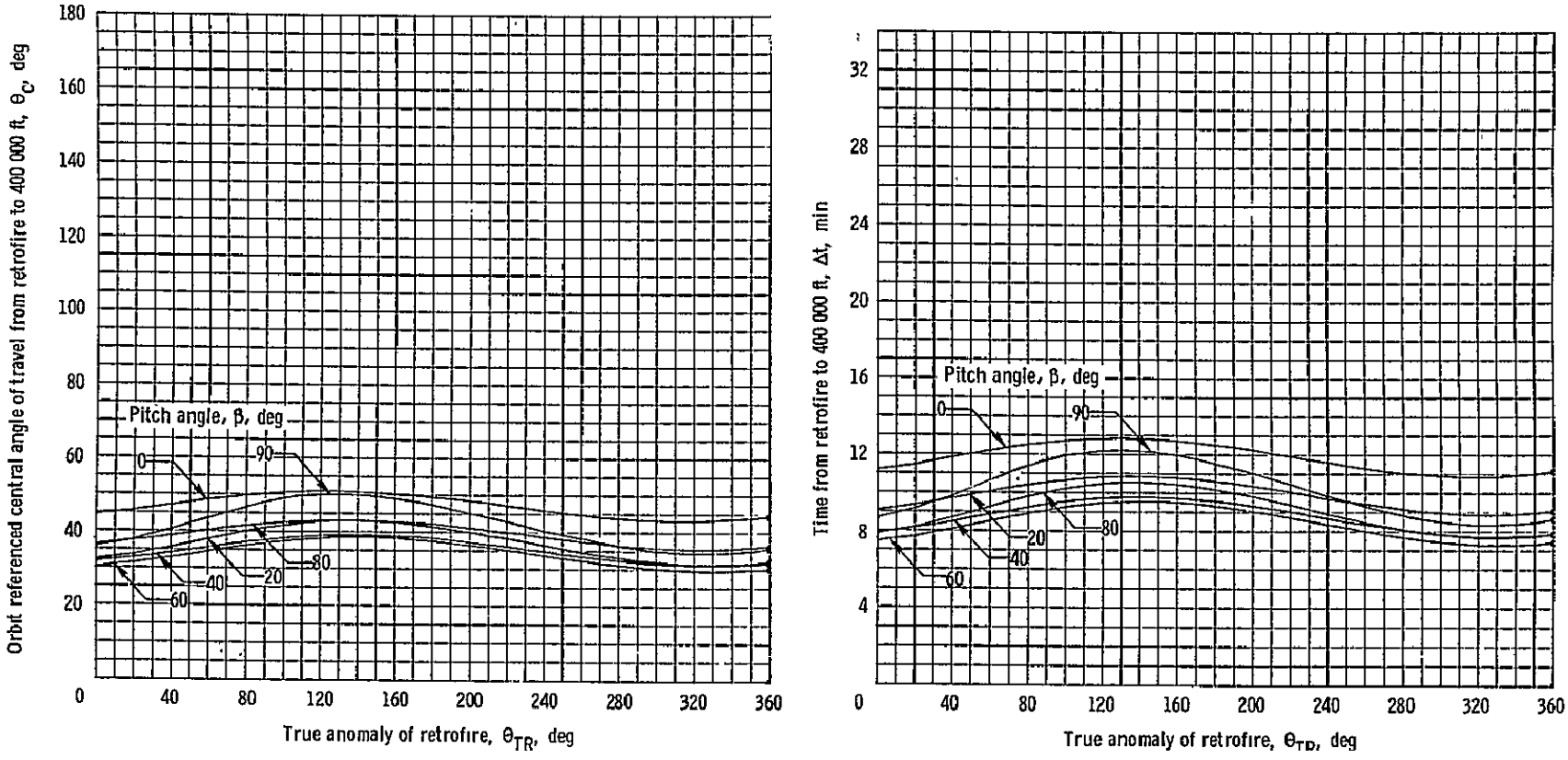
(b) Retrograde ΔV = 300 fps.

Figure 18. - Continued.



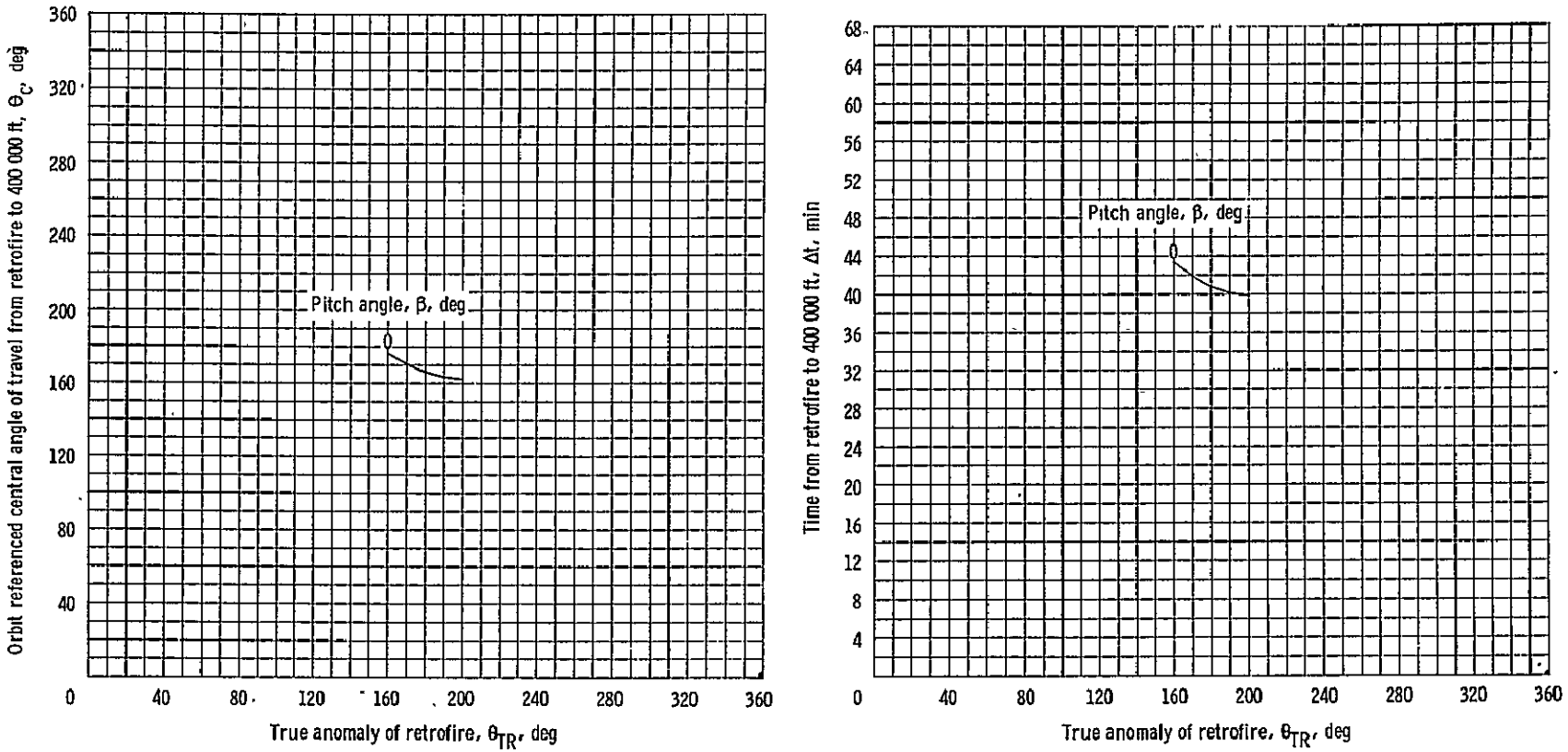
(c) Retrograde ΔV = 500 fps.

Figure 18. - Continued.



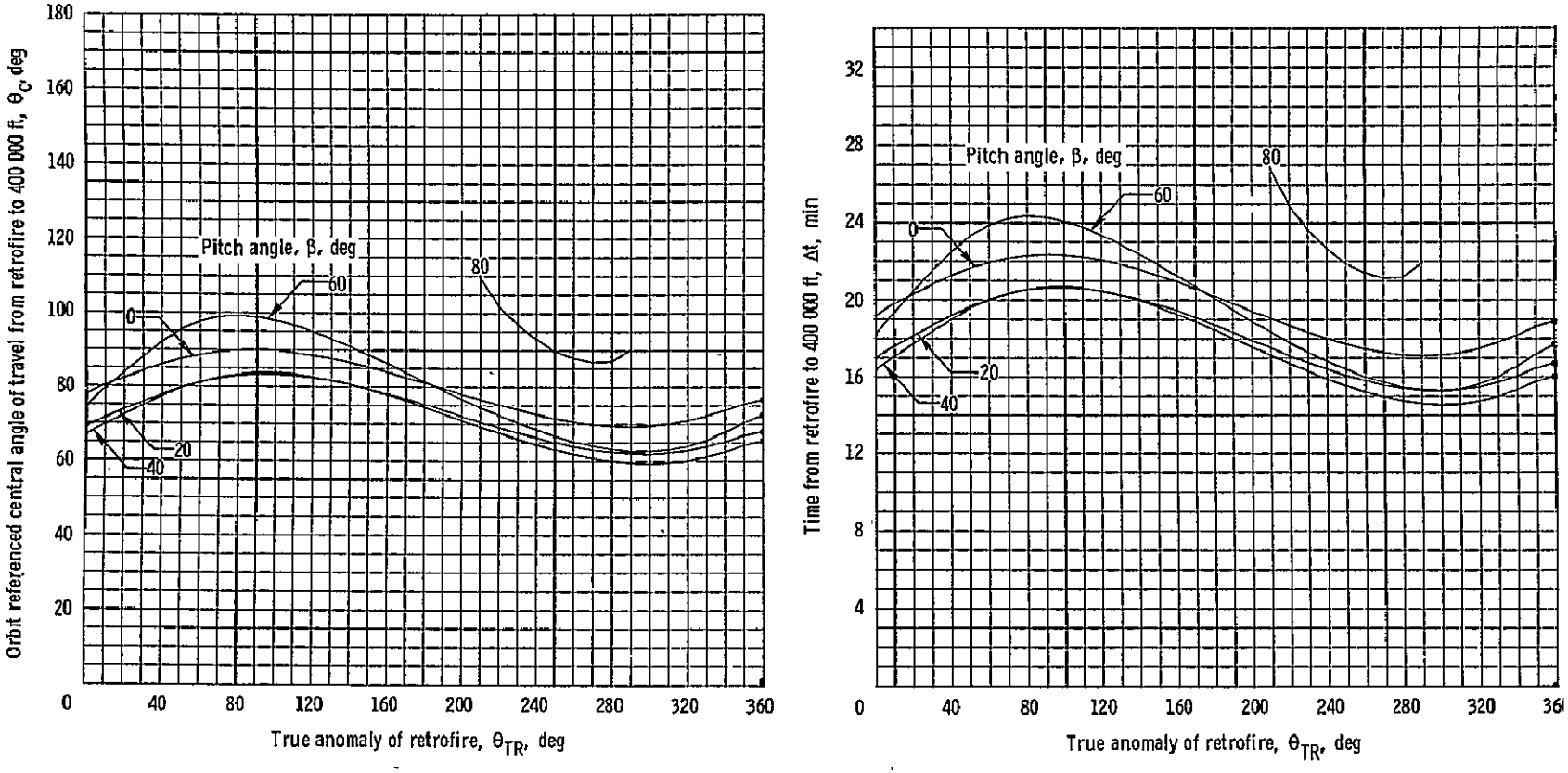
(d) Retrograde ΔV = 700 fps.

Figure 18. - Concluded.



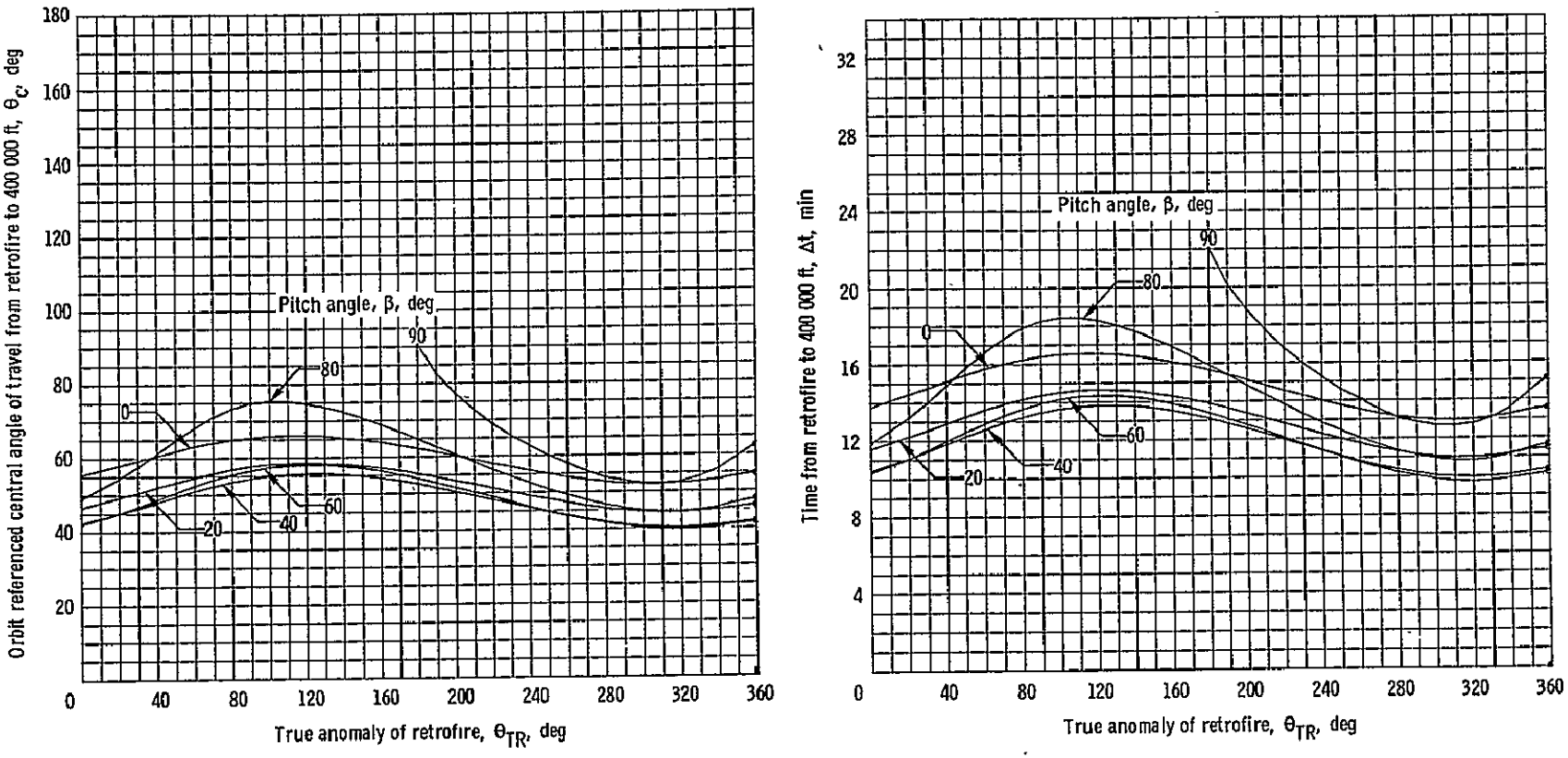
(a) Retrograde ΔV = 100 fps.

Figure 19. – Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 150 nautical miles and h_p = 120 nautical miles.



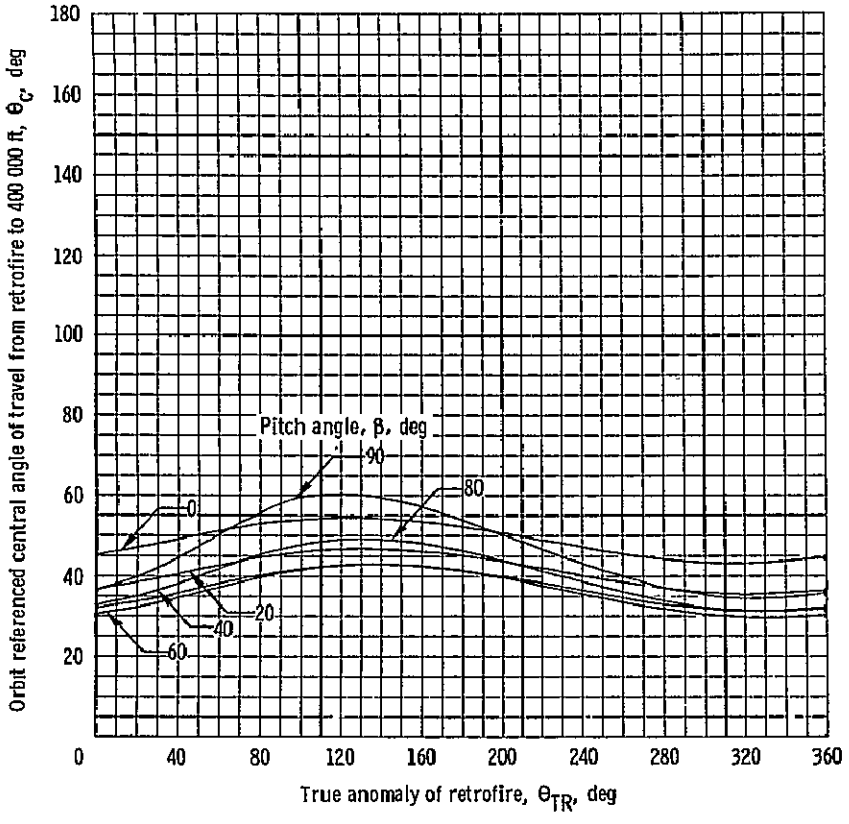
(b) Retrograde ΔV = 300 fps.

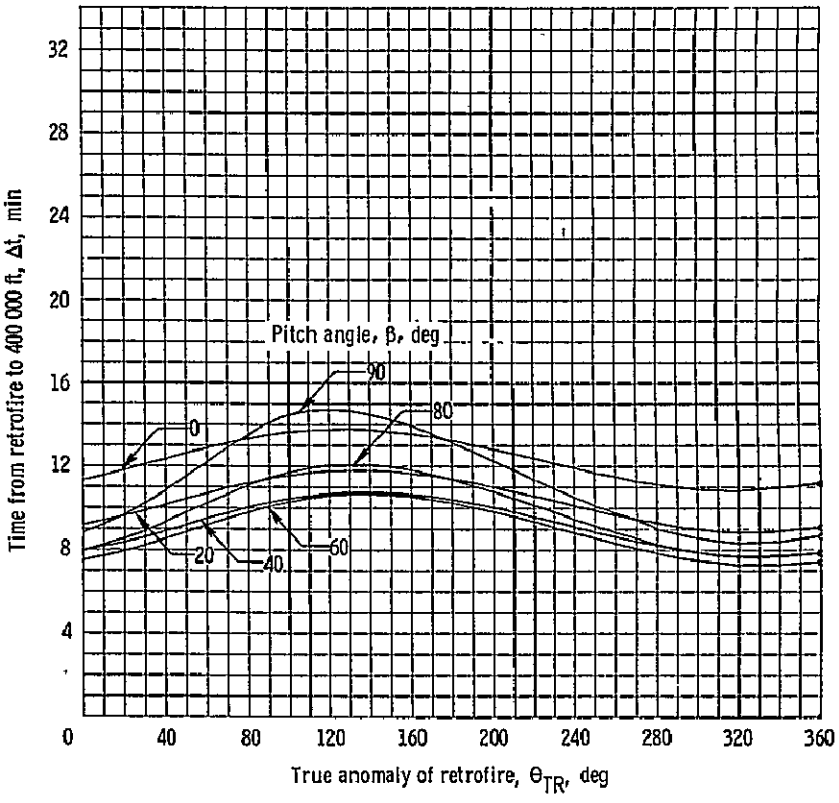
Figure 19. - Continued.



(c) Retrograde ΔV = 500 fps.

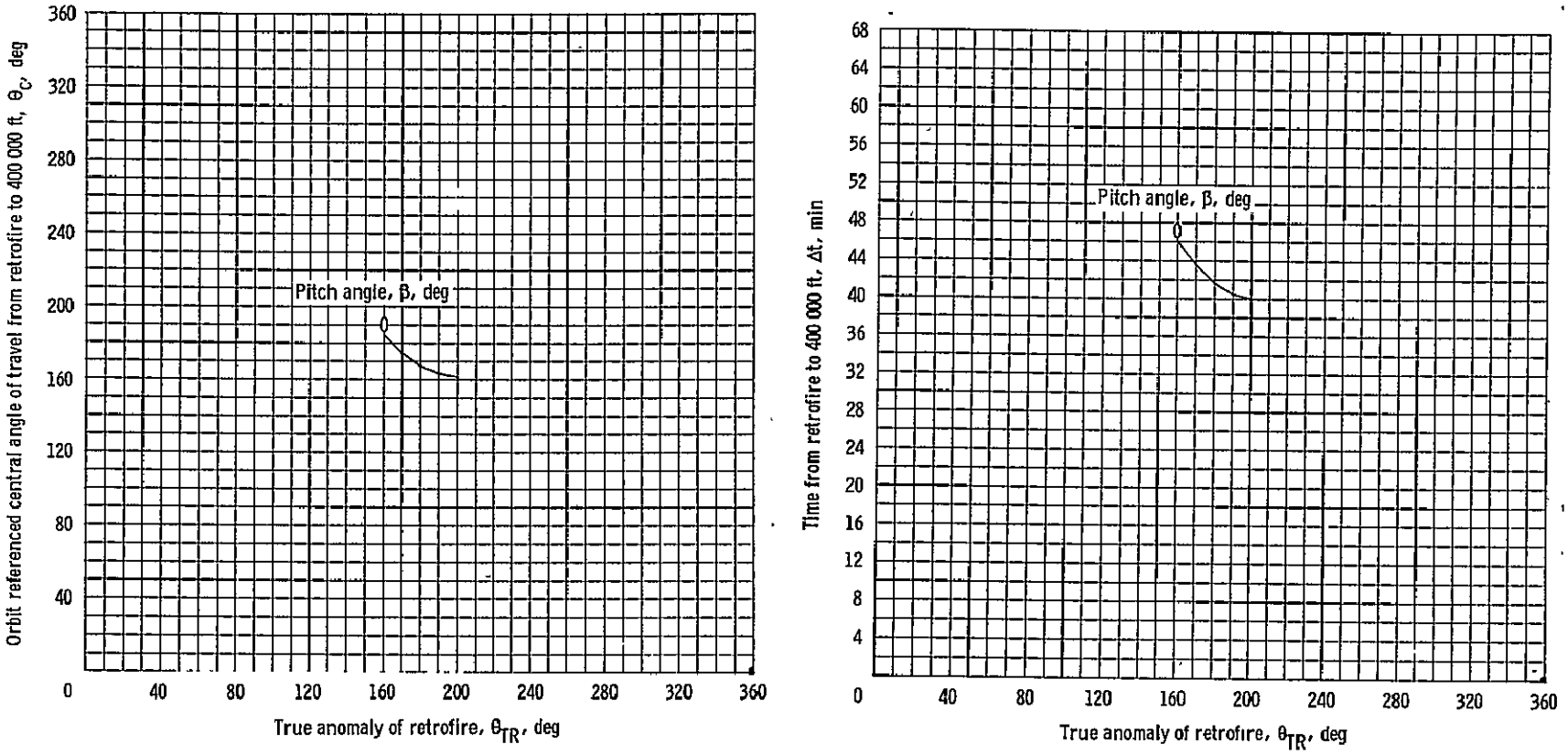
Figure 19. - Continued.





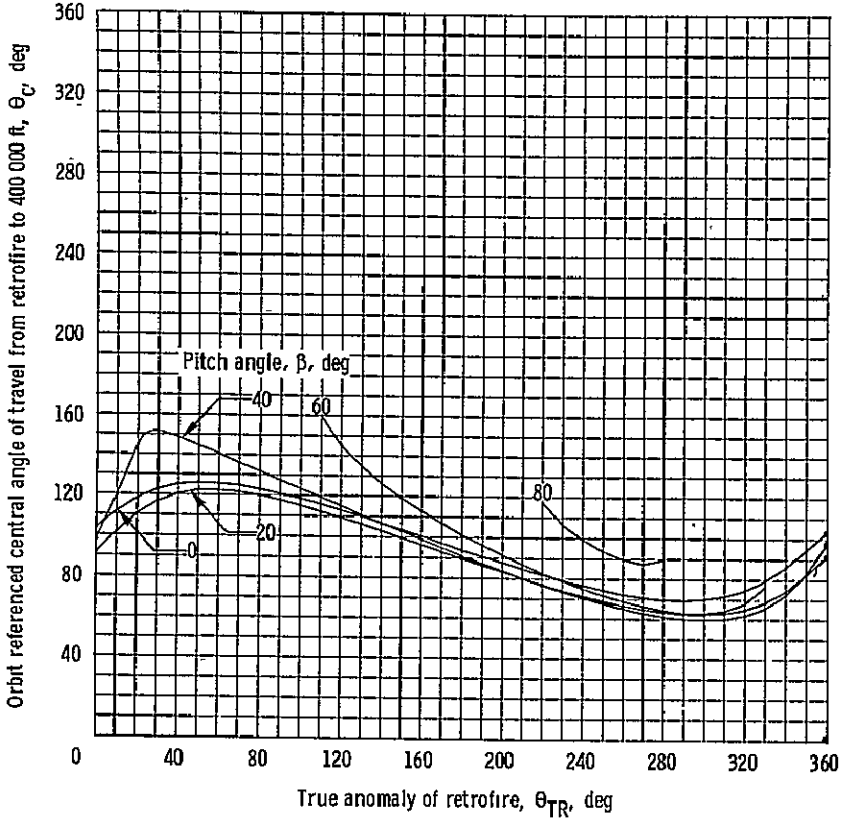
(d) Retrograde ΔV = 700 fps.

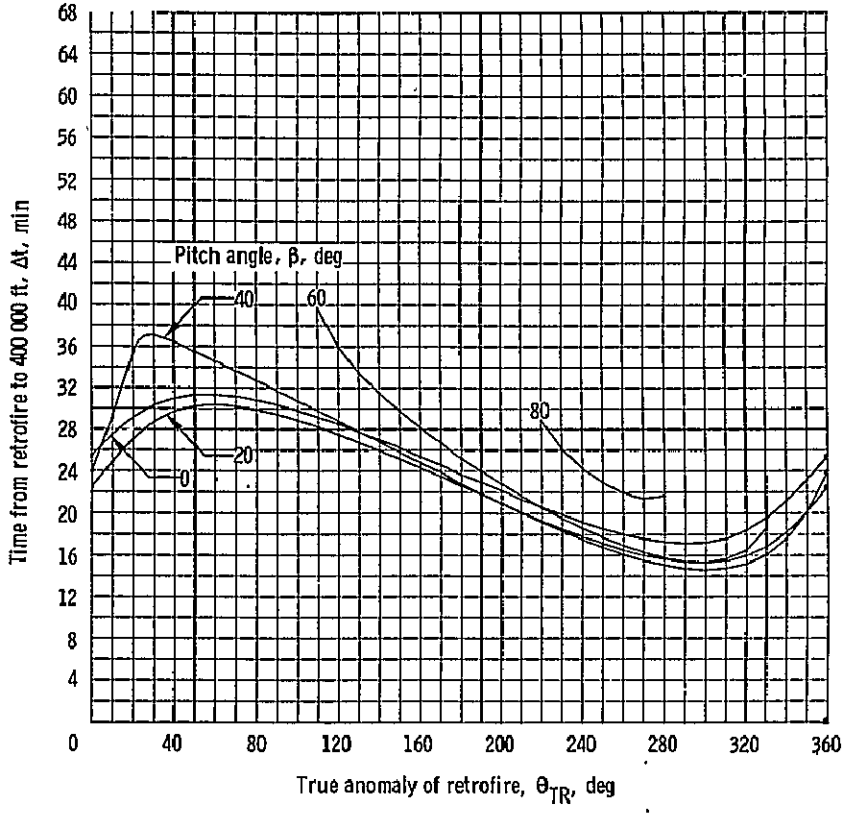
Figure 19.- Concluded.



(a) Retrograde ΔV = 100 fps.

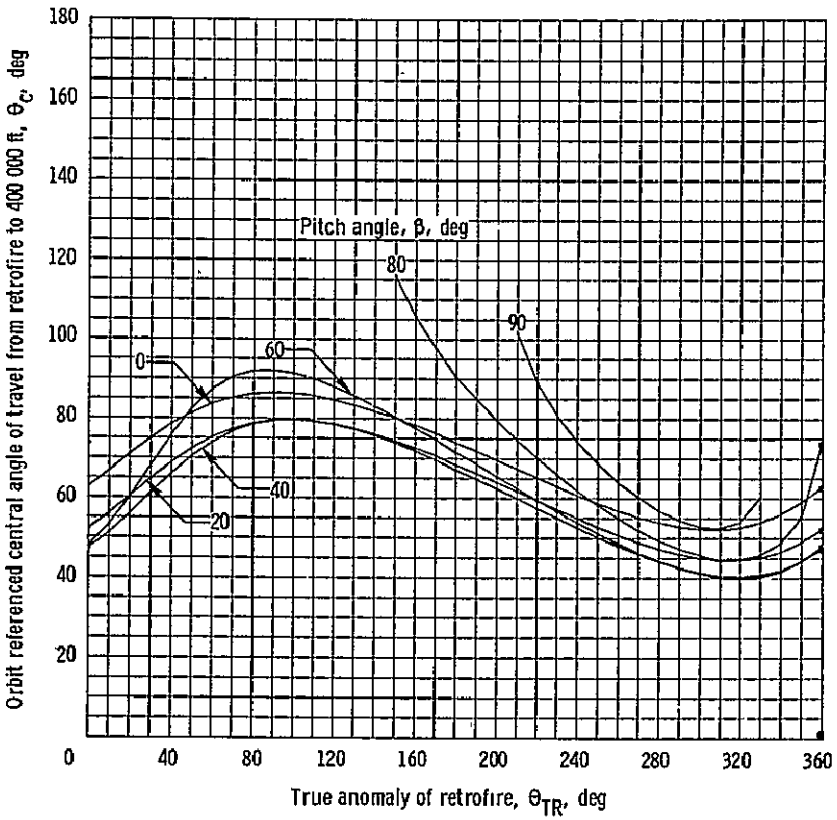
Figure 20.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 200 nautical miles and h_p = 120 nautical miles.

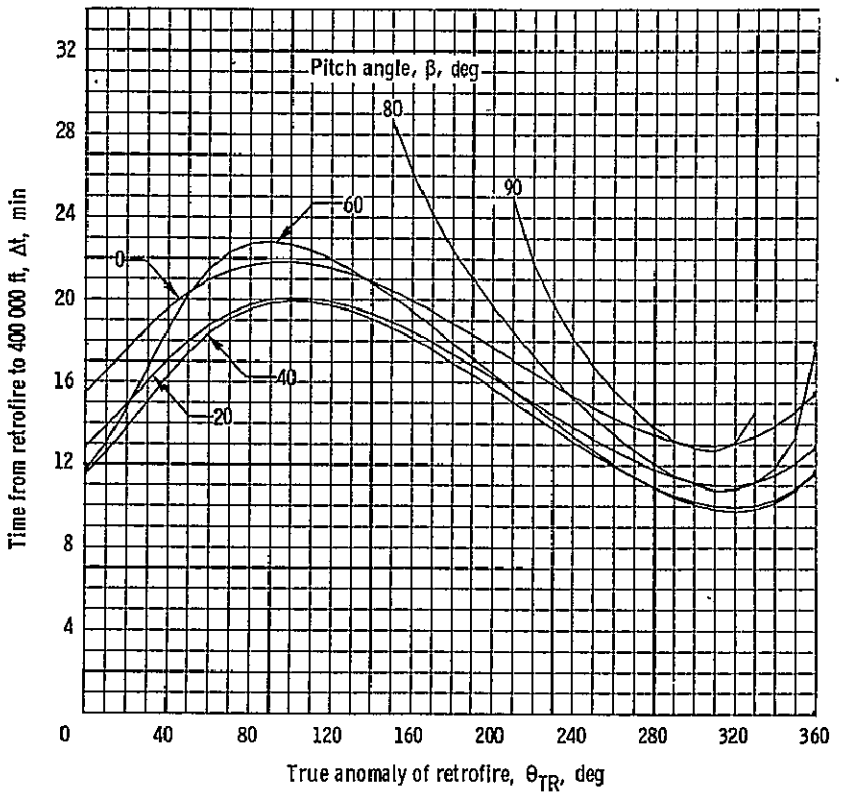




(b) Retrograde ΔV = 300 fps.

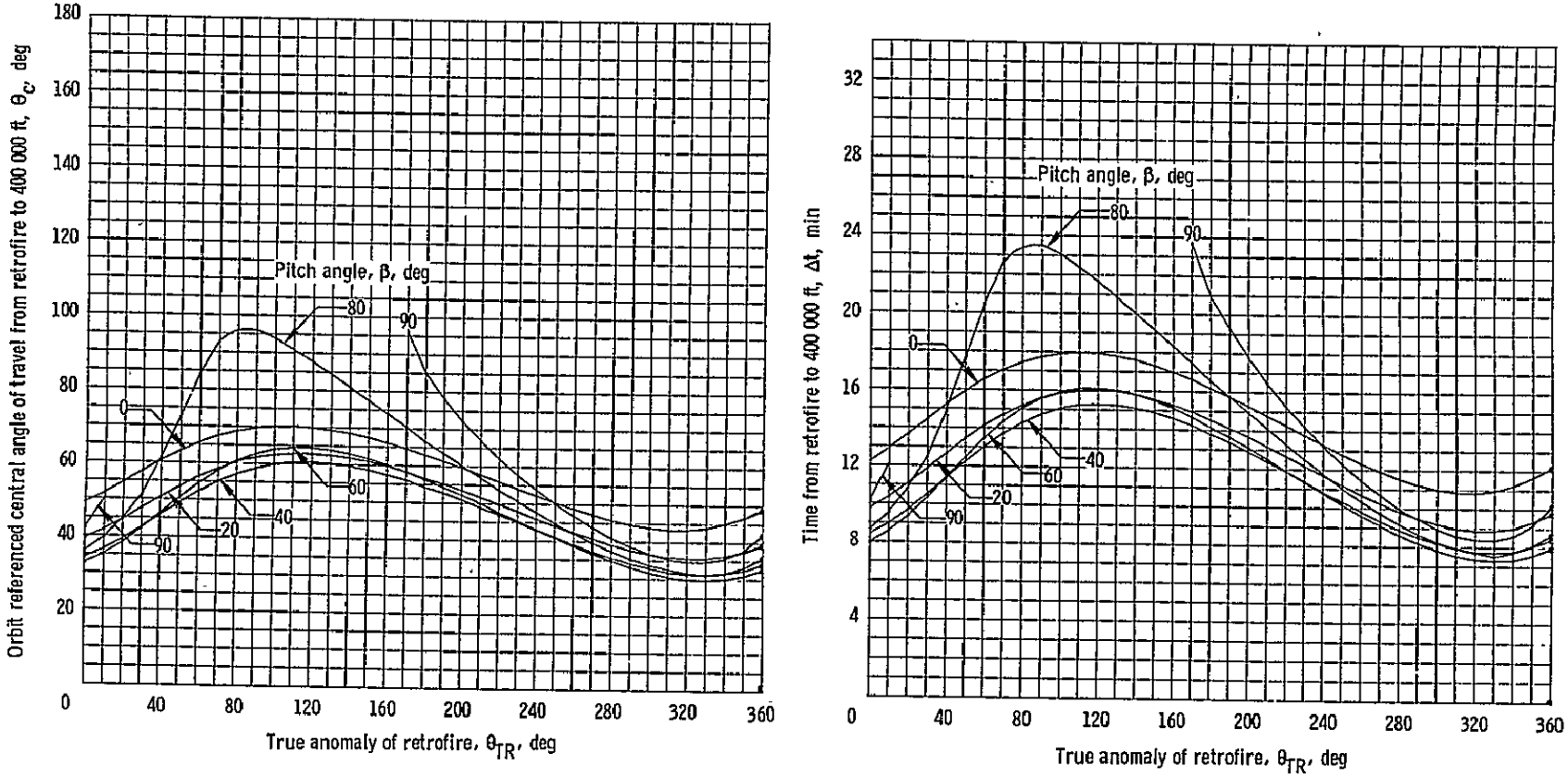
Figure 20.- Continued.





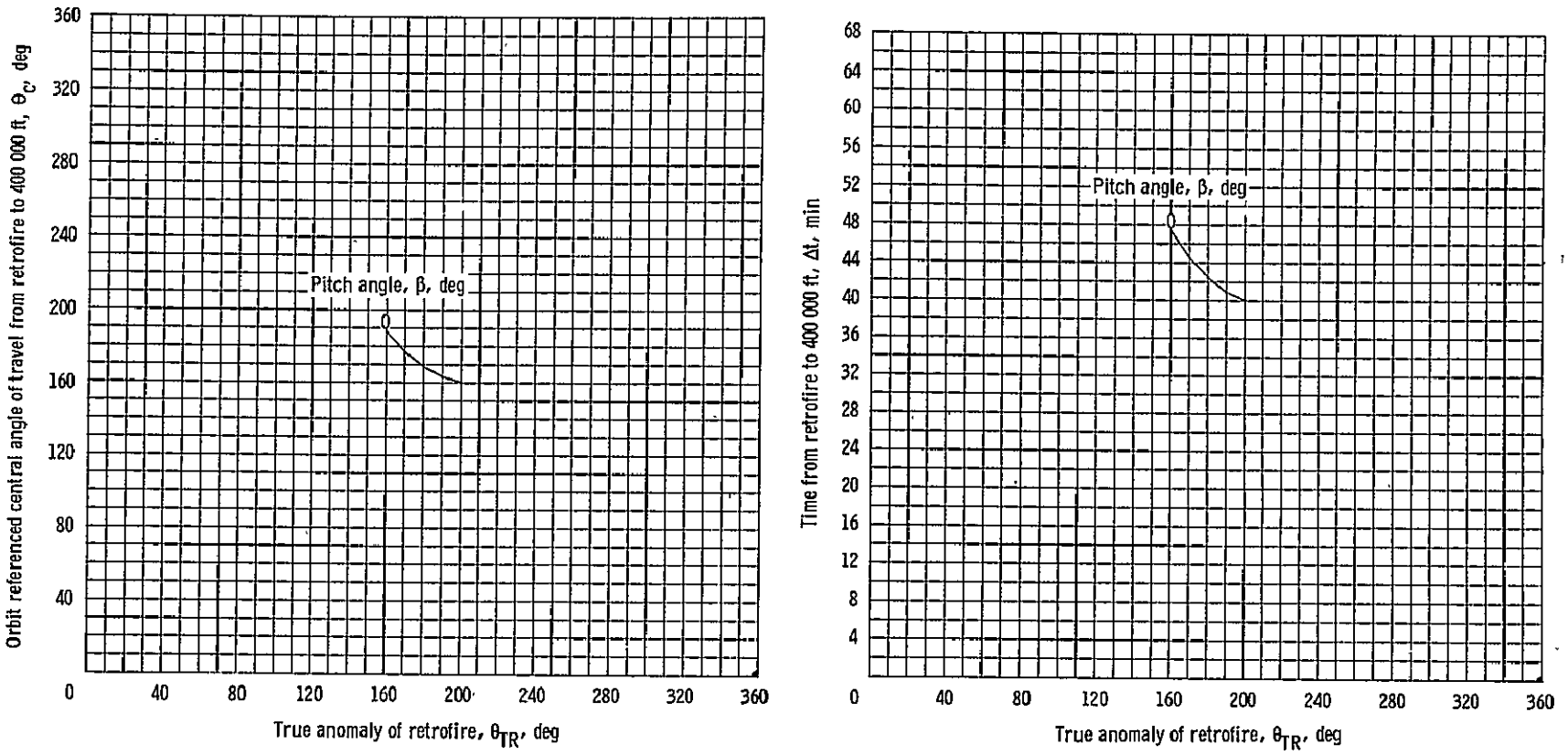
(c) Retrograde ΔV = 500 fps.

Figure 20. - Continued.



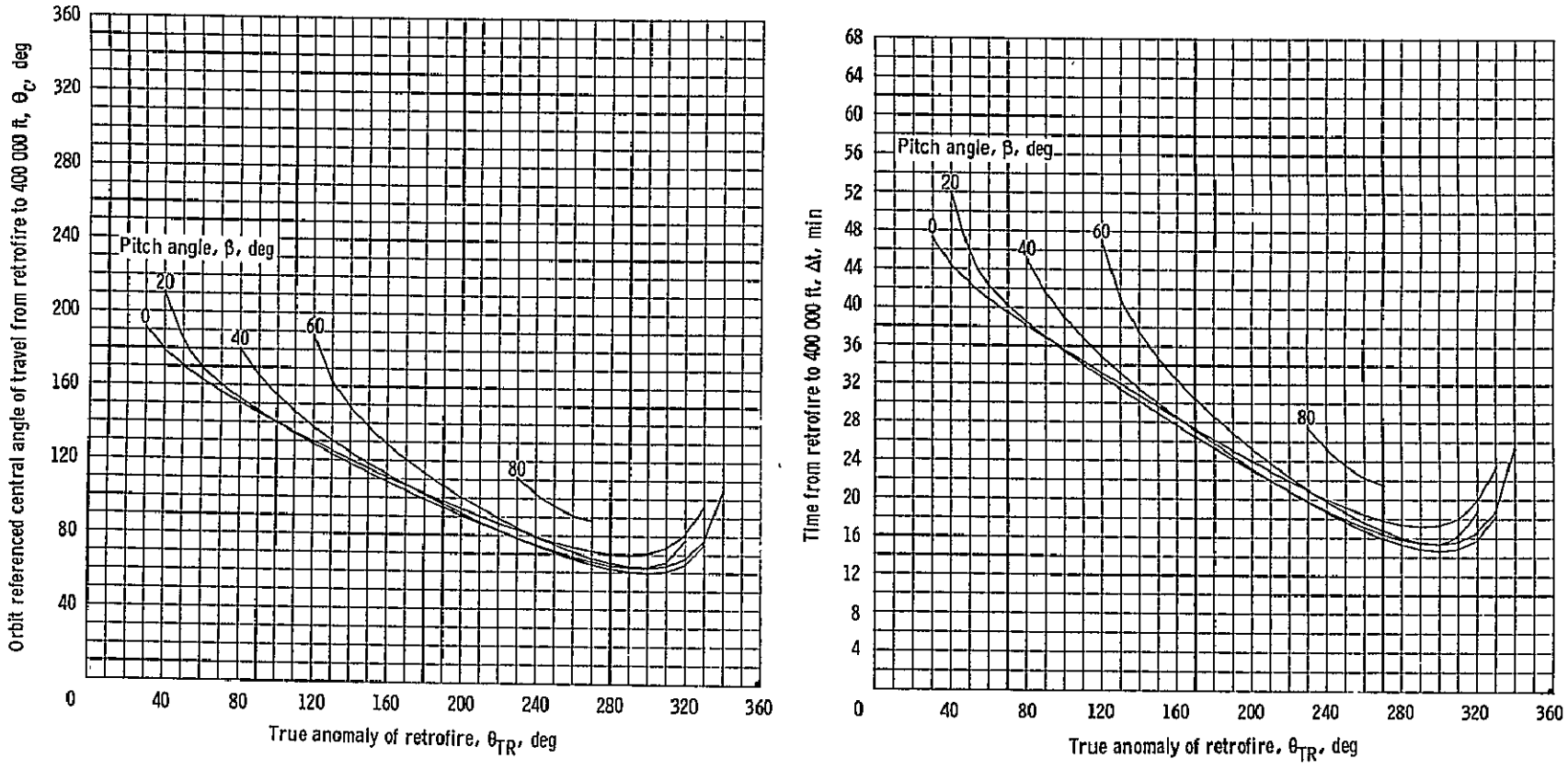
(d) Retrograde ΔV = 700 fps.

Figure 20. - Concluded.



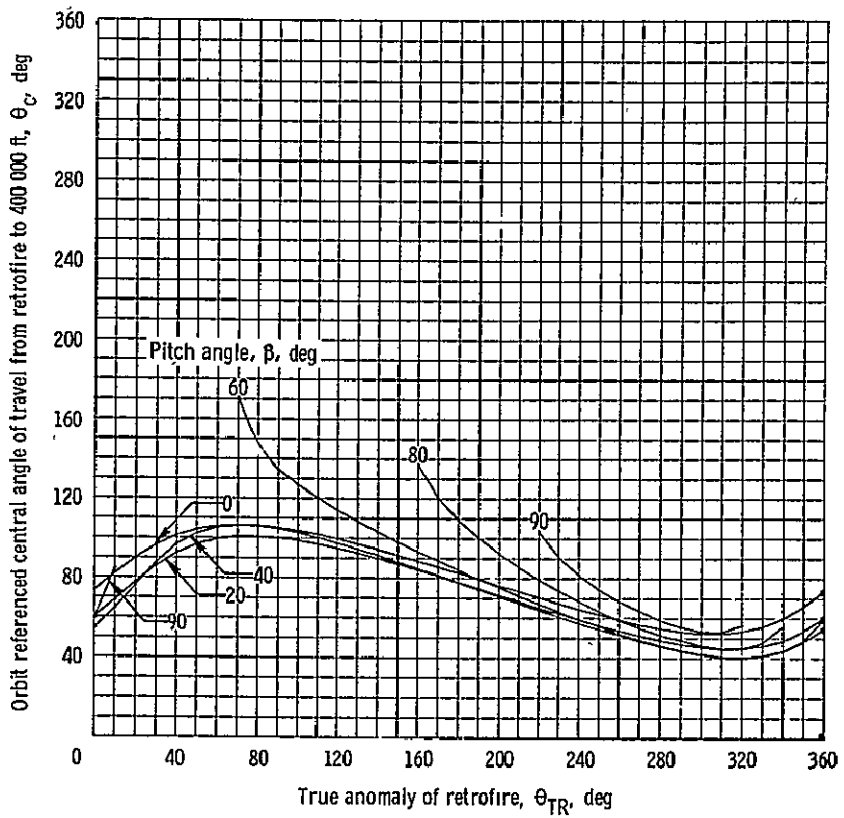
(a) Retrograde ΔV = 100 fps.

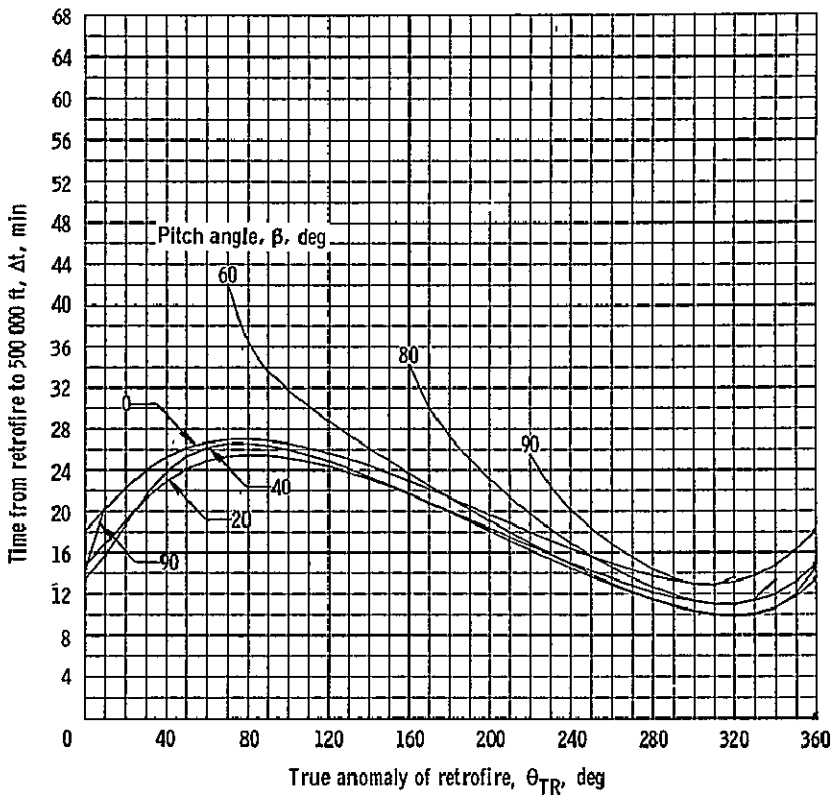
Figure 21. - Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 250 nautical miles and h_p = 120 nautical miles.



(b) Retrograde ΔV = 300 fps.

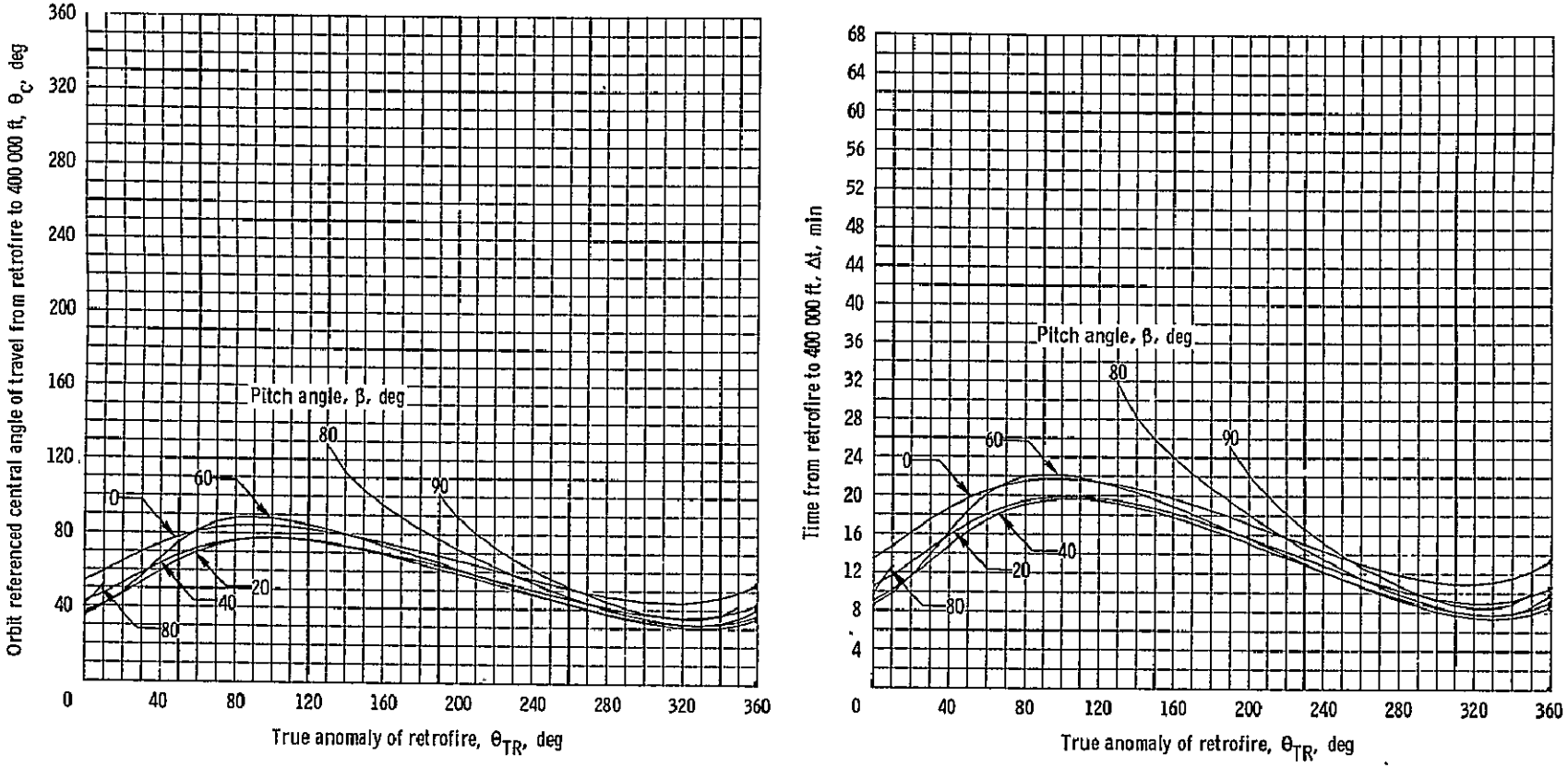
Figure 21. - Continued.





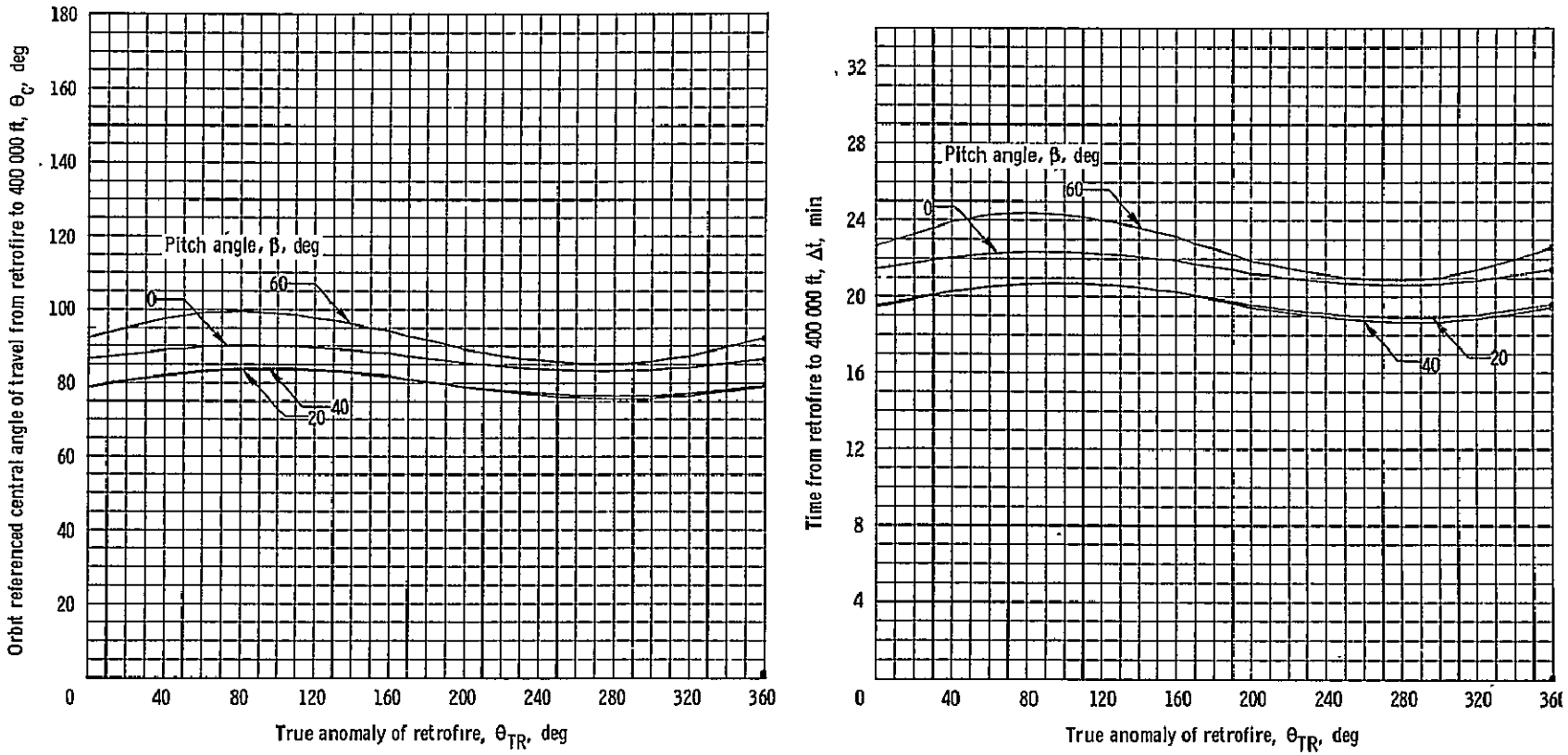
(c) Retrograde ΔV = 500 fps.

Figure 21.- Continued.



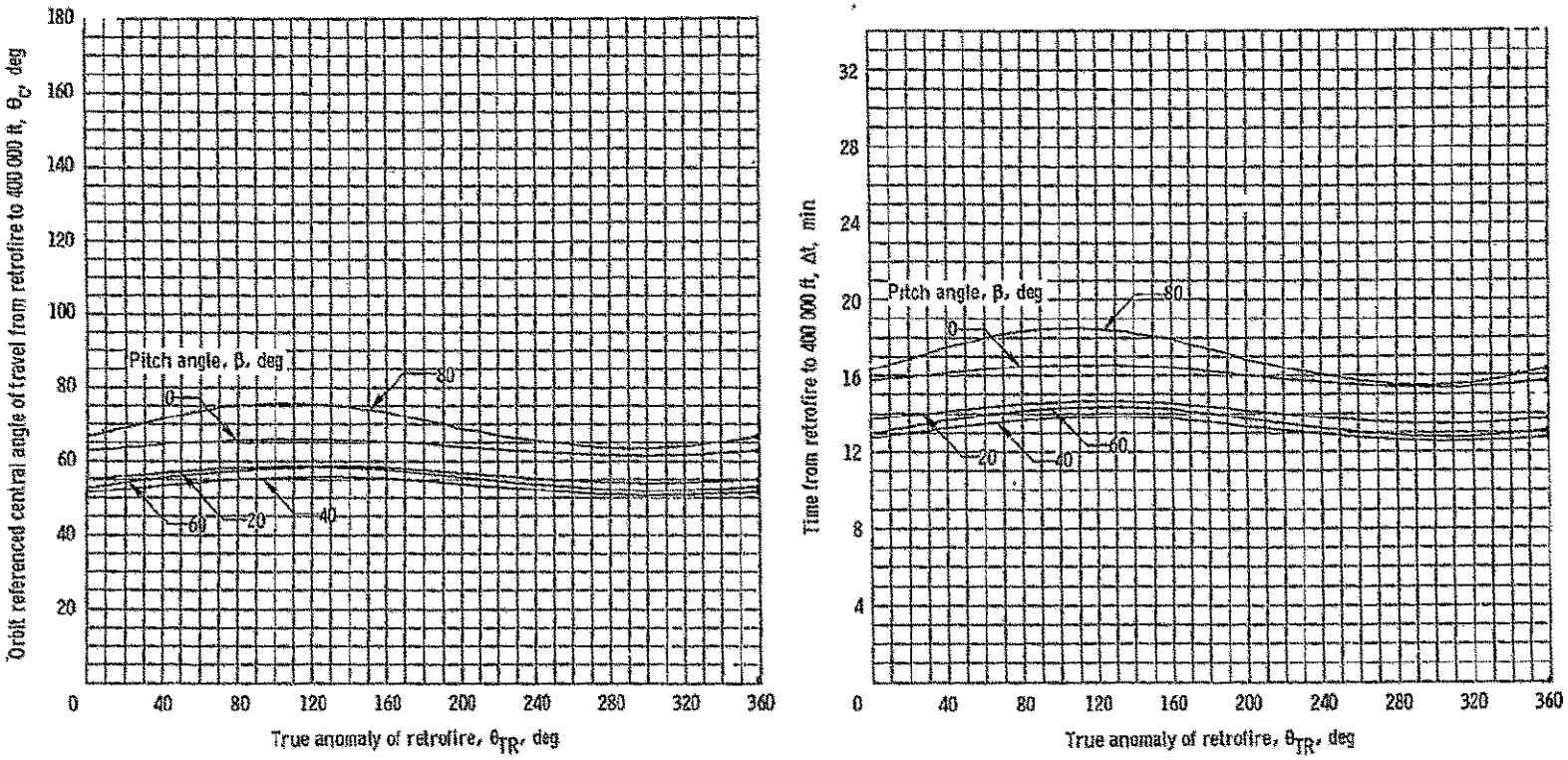
(d) Retrograde ΔV = 700 fps.

Figure 21.- Concluded.



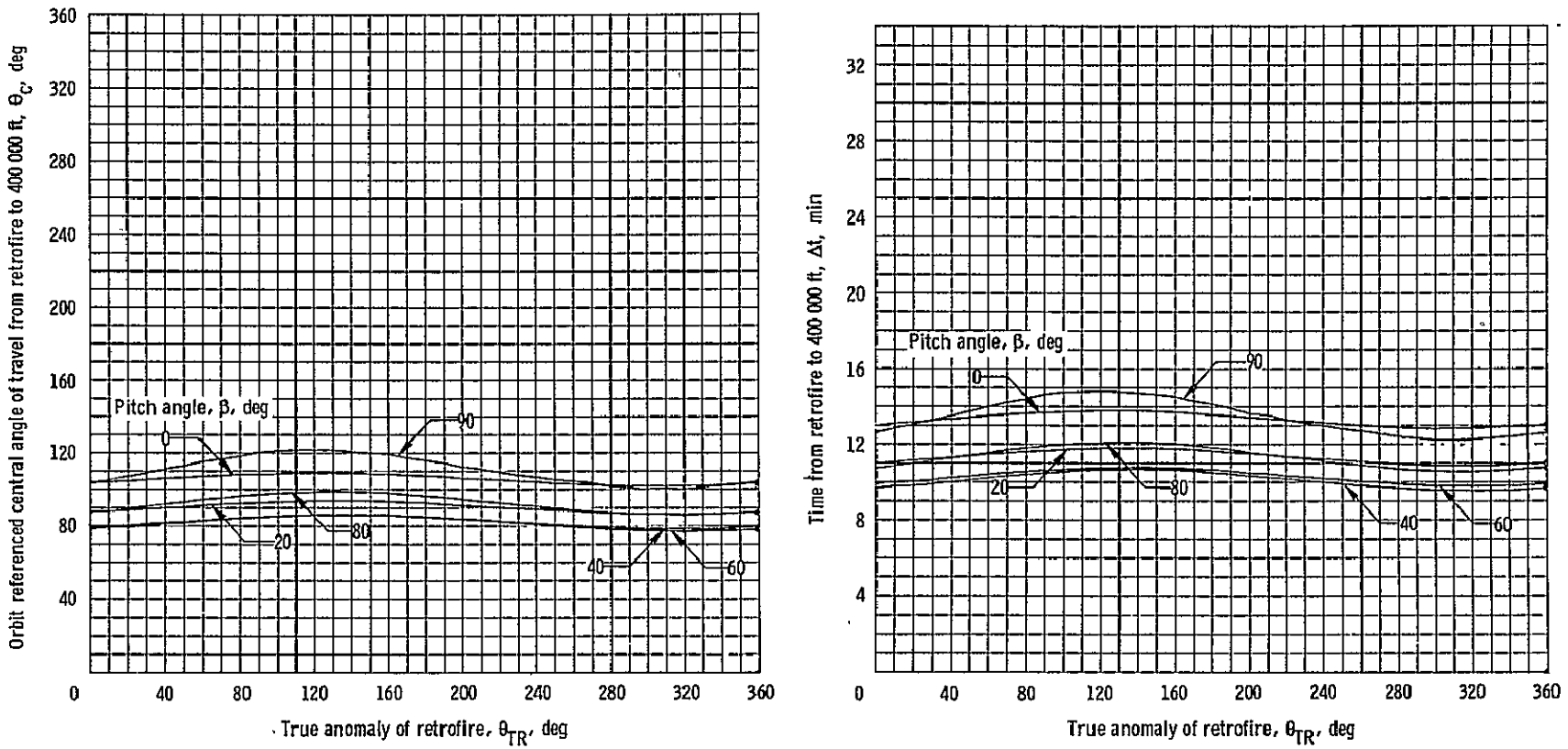
(a) Retrograde ΔV = 300 fps.

Figure 22.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 150 nautical miles and h_p = 140 nautical miles.



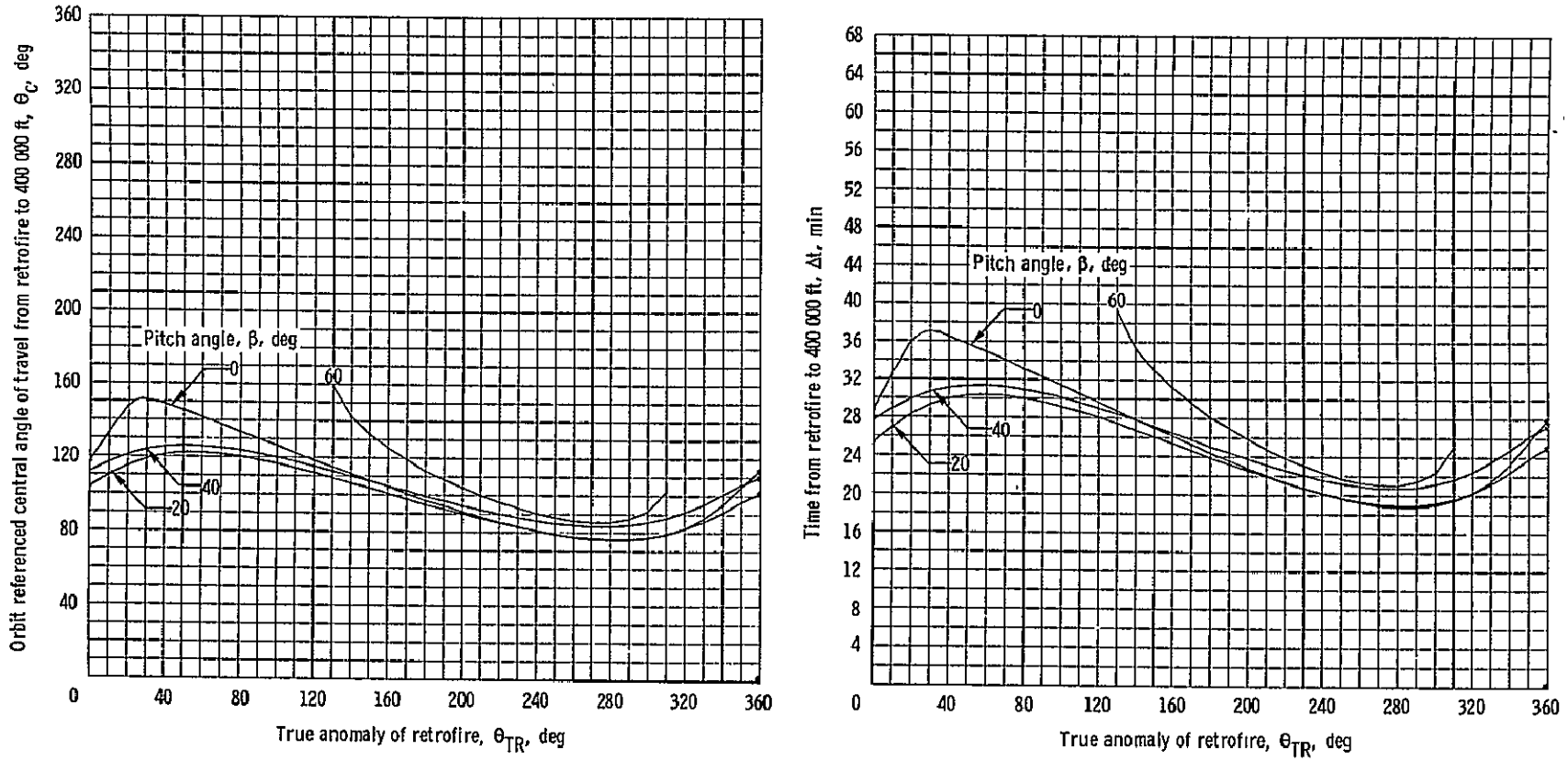
(b) Retrograde ΔV = 500 fps.

Figure 22. - Continued.



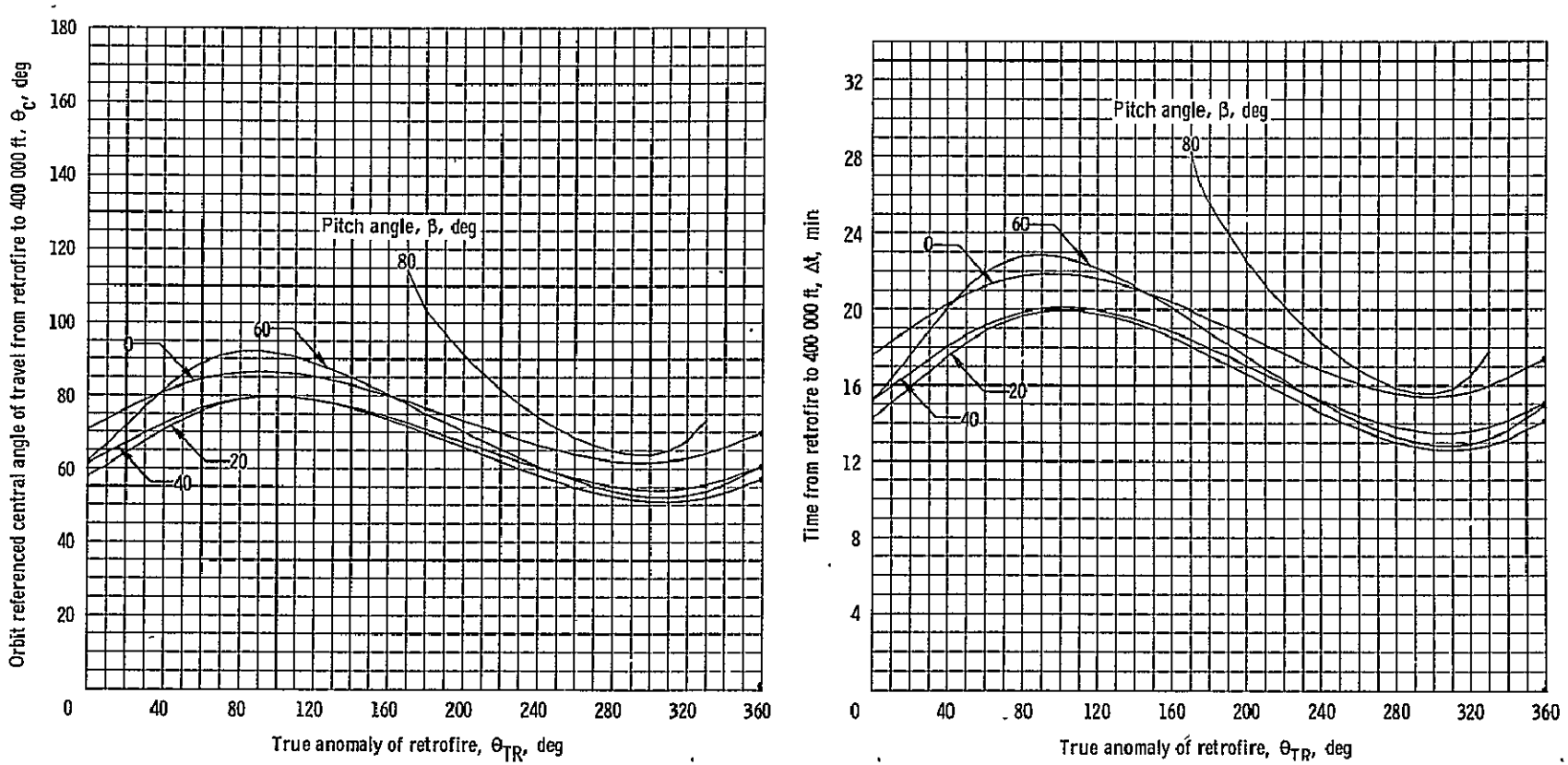
(c) Retrograde ΔV = 700 fps.

Figure 22. - Concluded.



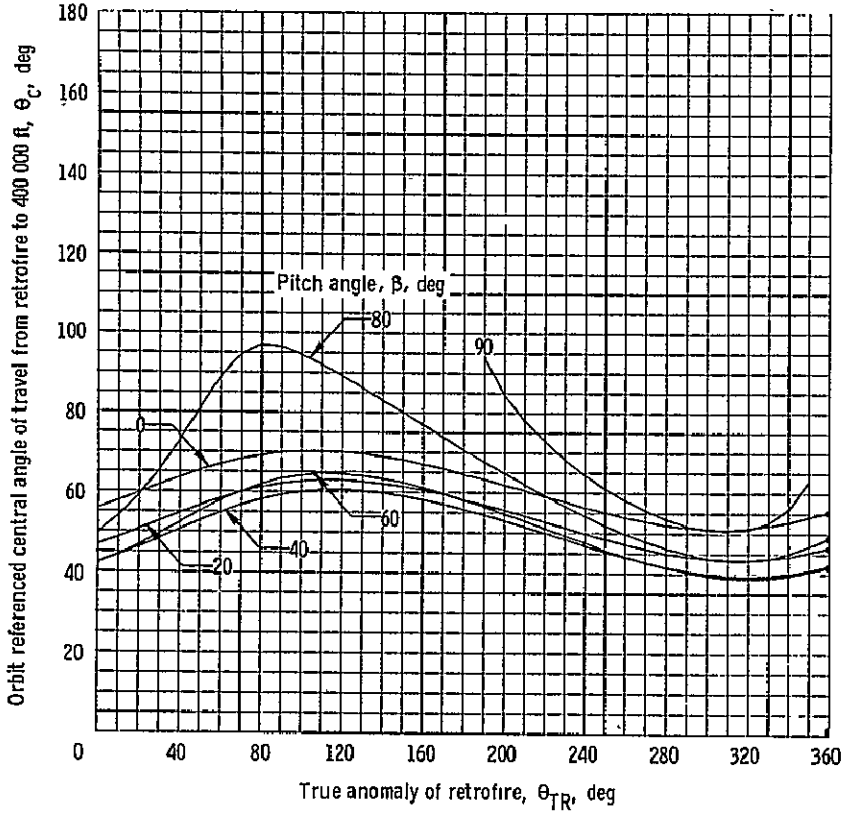
(a) Retrograde ΔV = 300 fps.

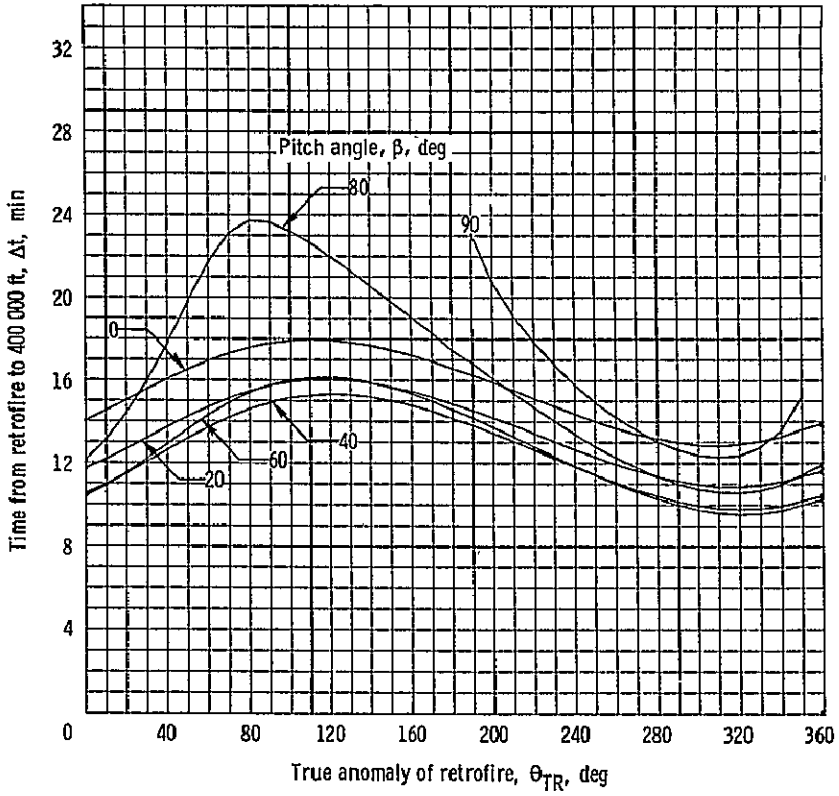
Figure 23.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 200 nautical miles and h_p = 140 nautical miles.



(b) Retrograde ΔV = 500 fps.

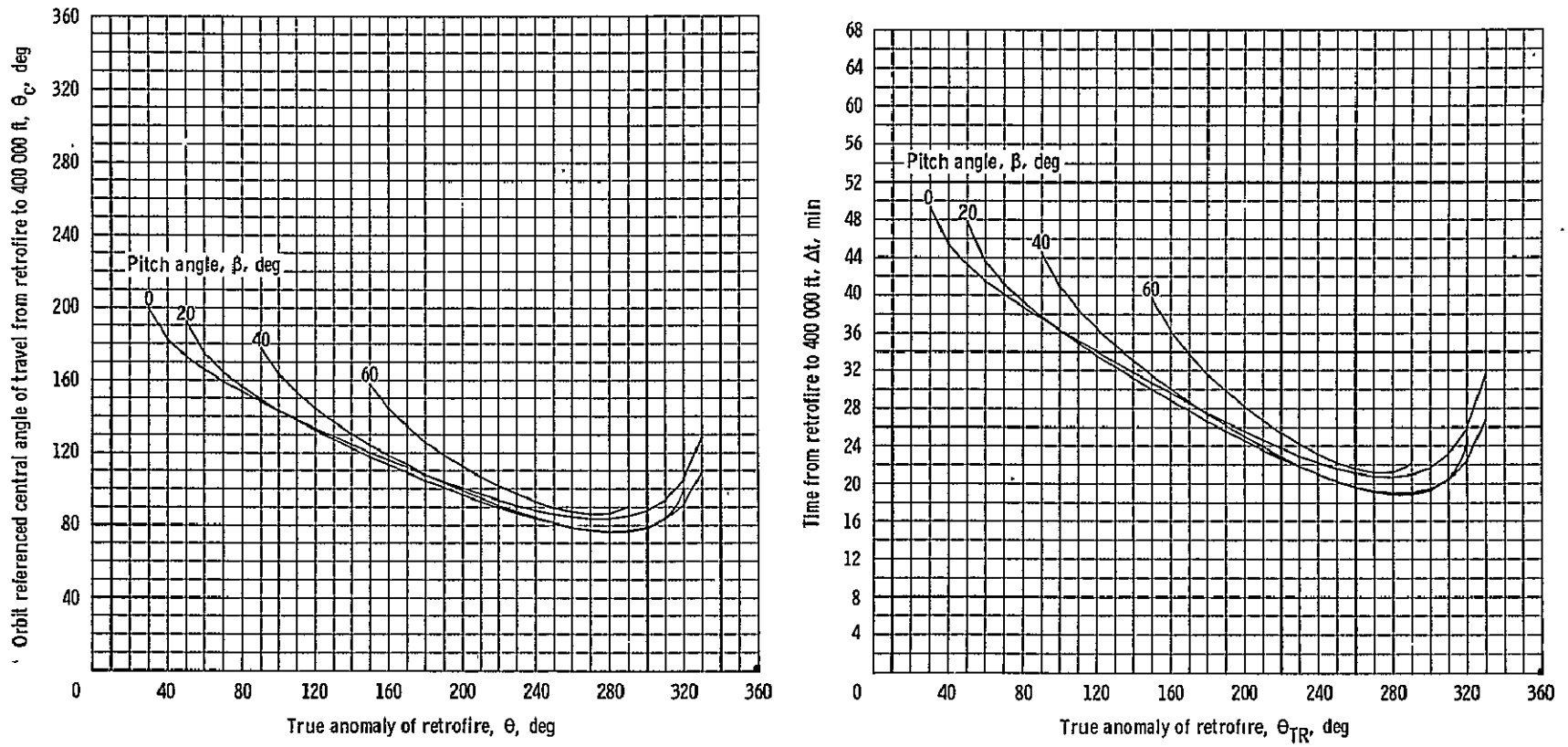
Figure 23.- Continued.





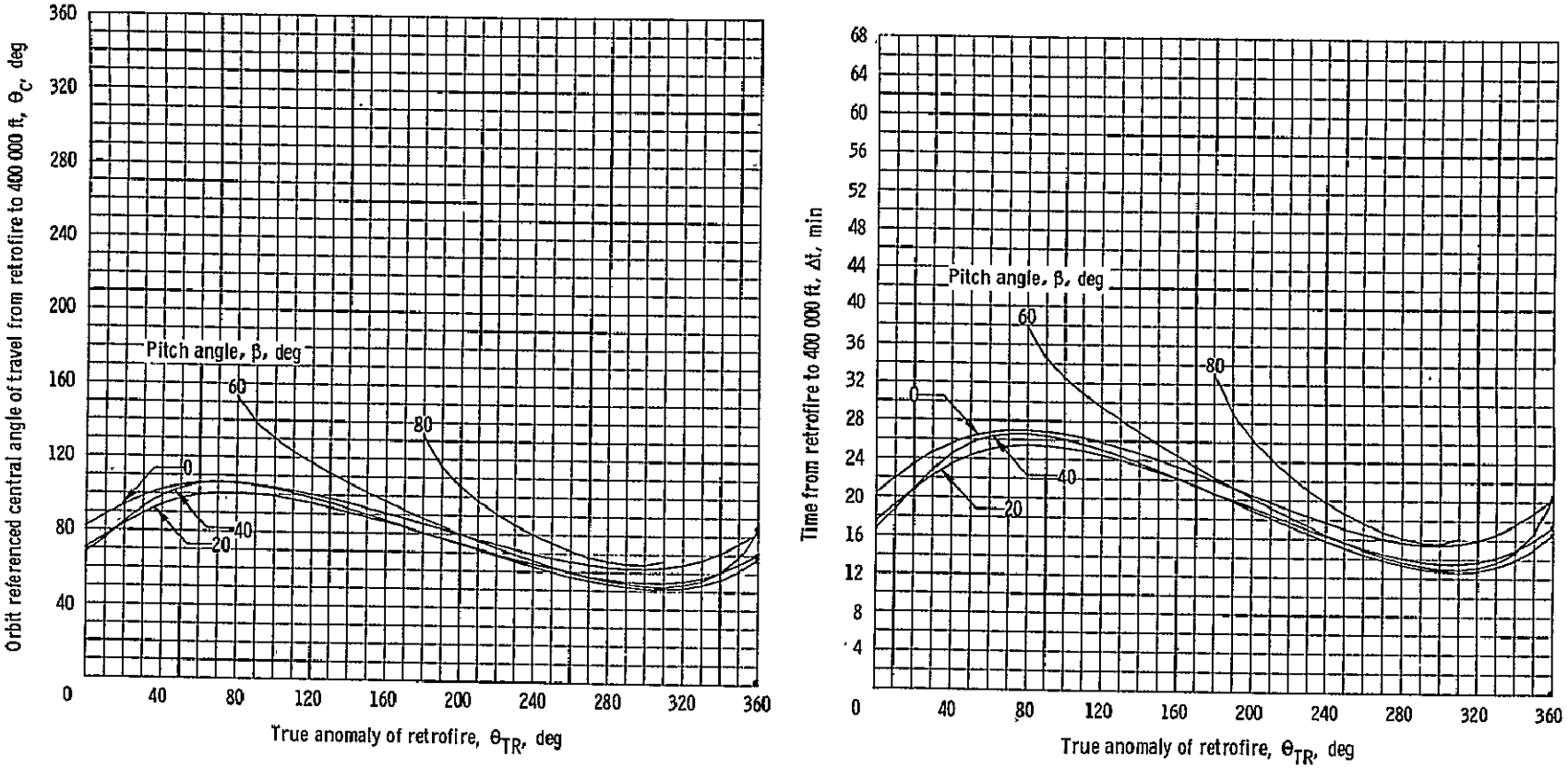
(c) Retrograde ΔV = 700 fps.

Figure 23. – Concluded.



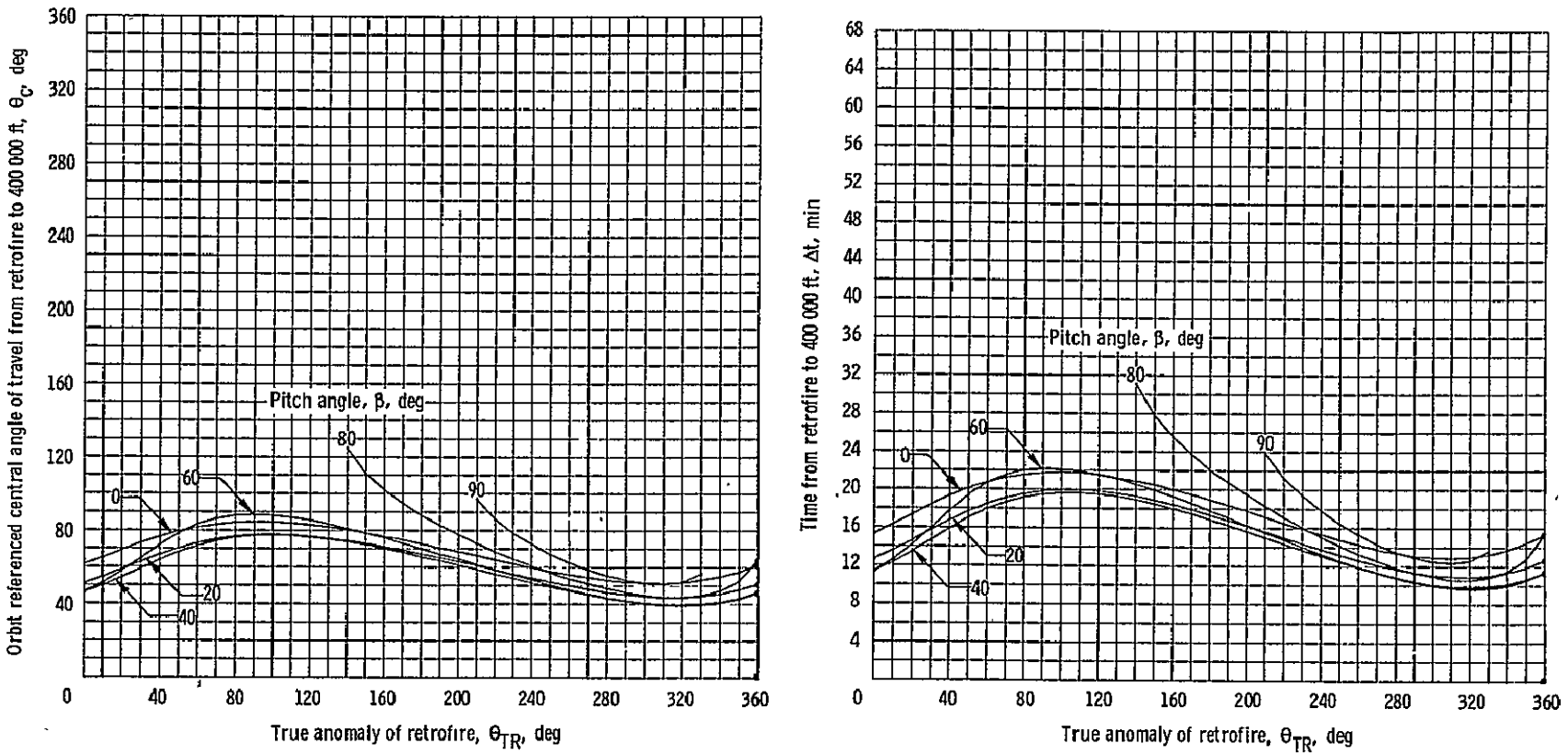
(a) Retrograde ΔV = 300 fps.

Figure 24. – Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 250 nautical miles and h_p = 140 nautical miles.



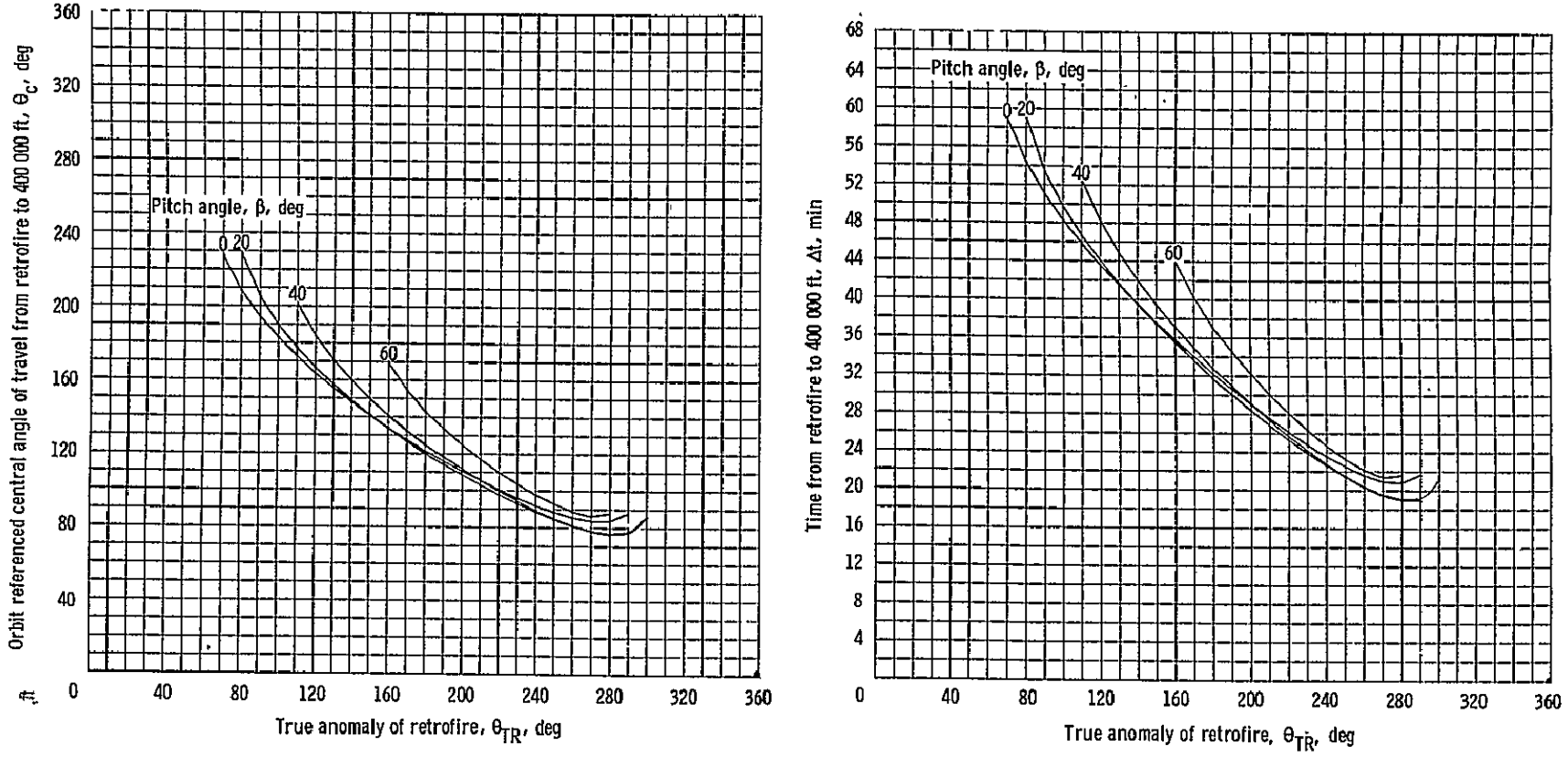
(b) Retrograde ΔV = 500 fps.

Figure 24. – Continued.



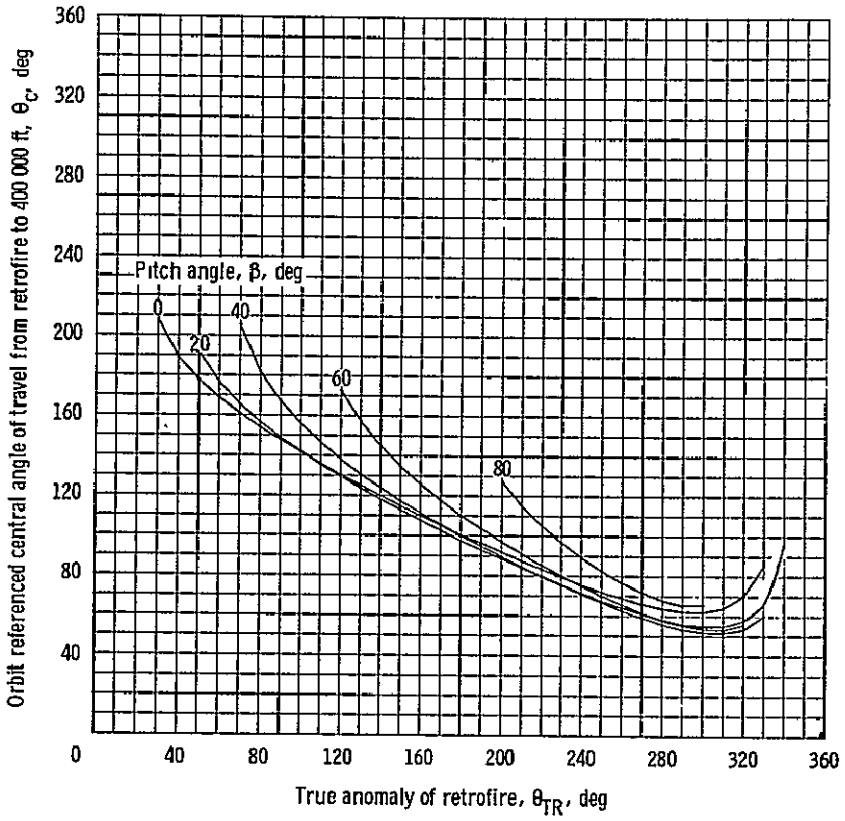
(c) Retrograde ΔV = 700 fps.

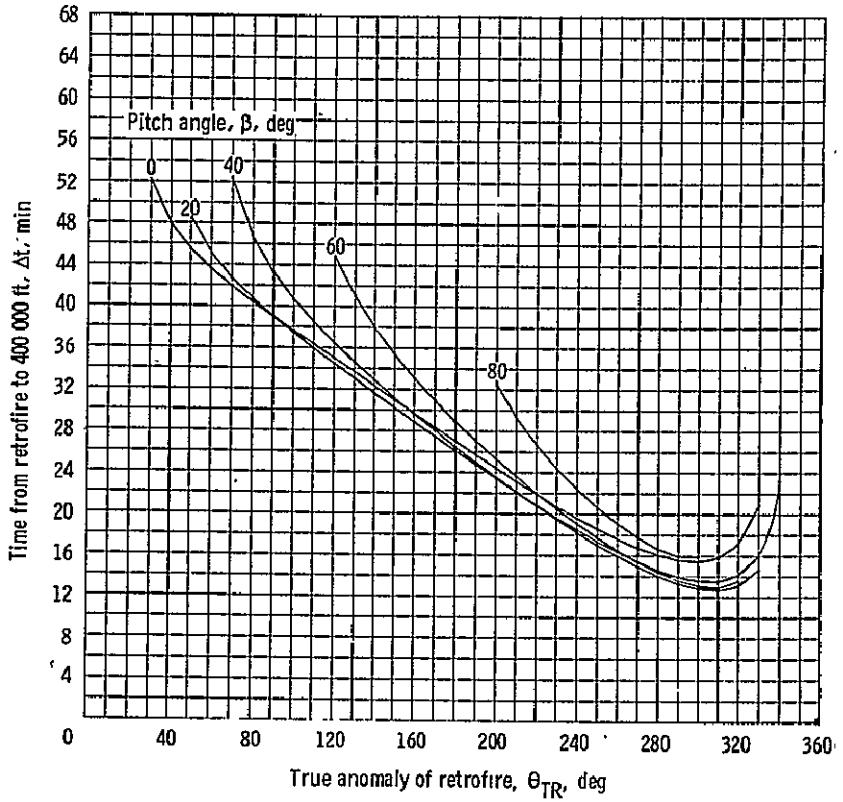
Figure 24. - Concluded.



(a) Retrograde ΔV = 300 fps.

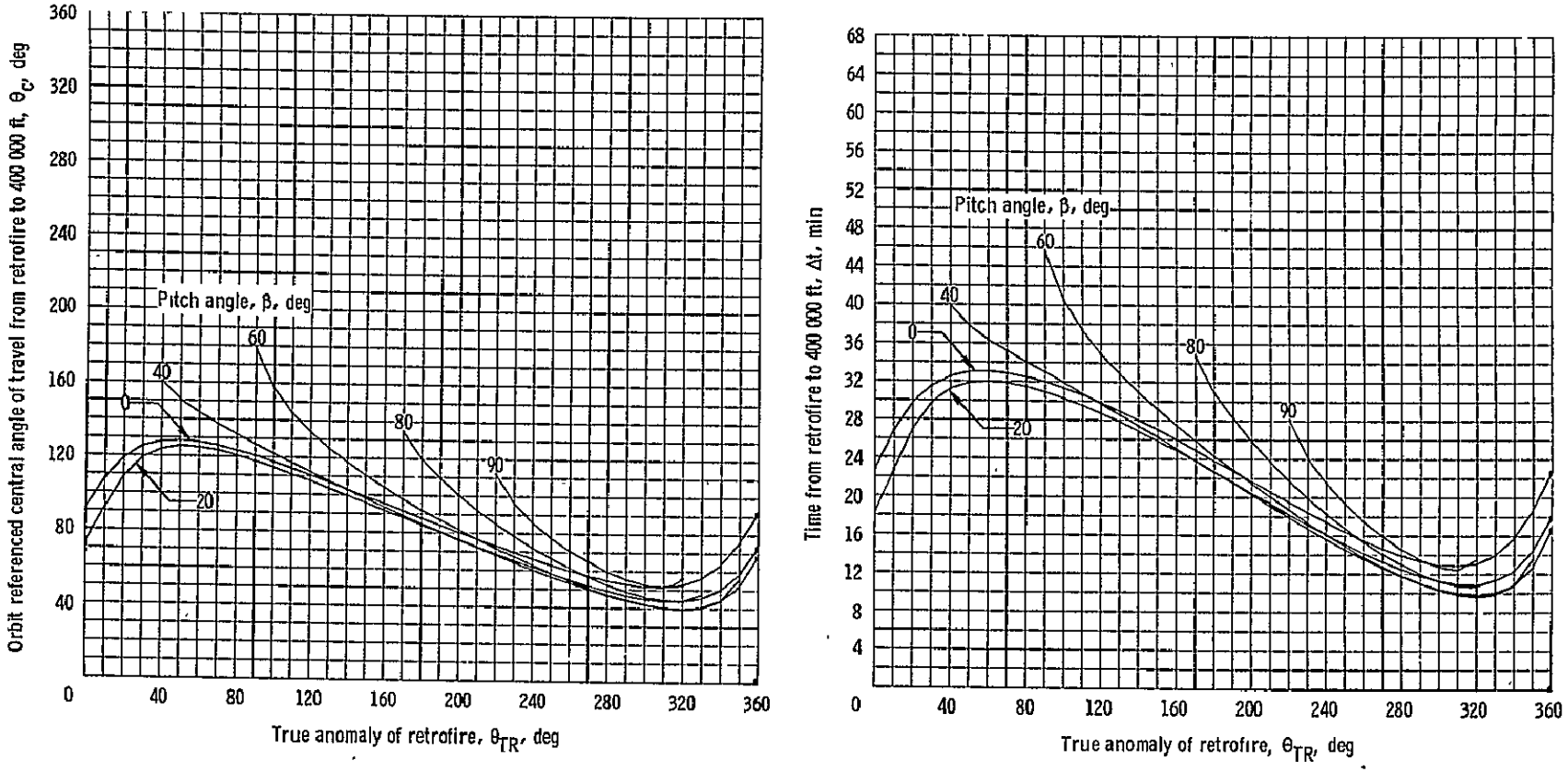
Figure 25. - Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 400 nautical miles and h_p = 140 nautical miles.





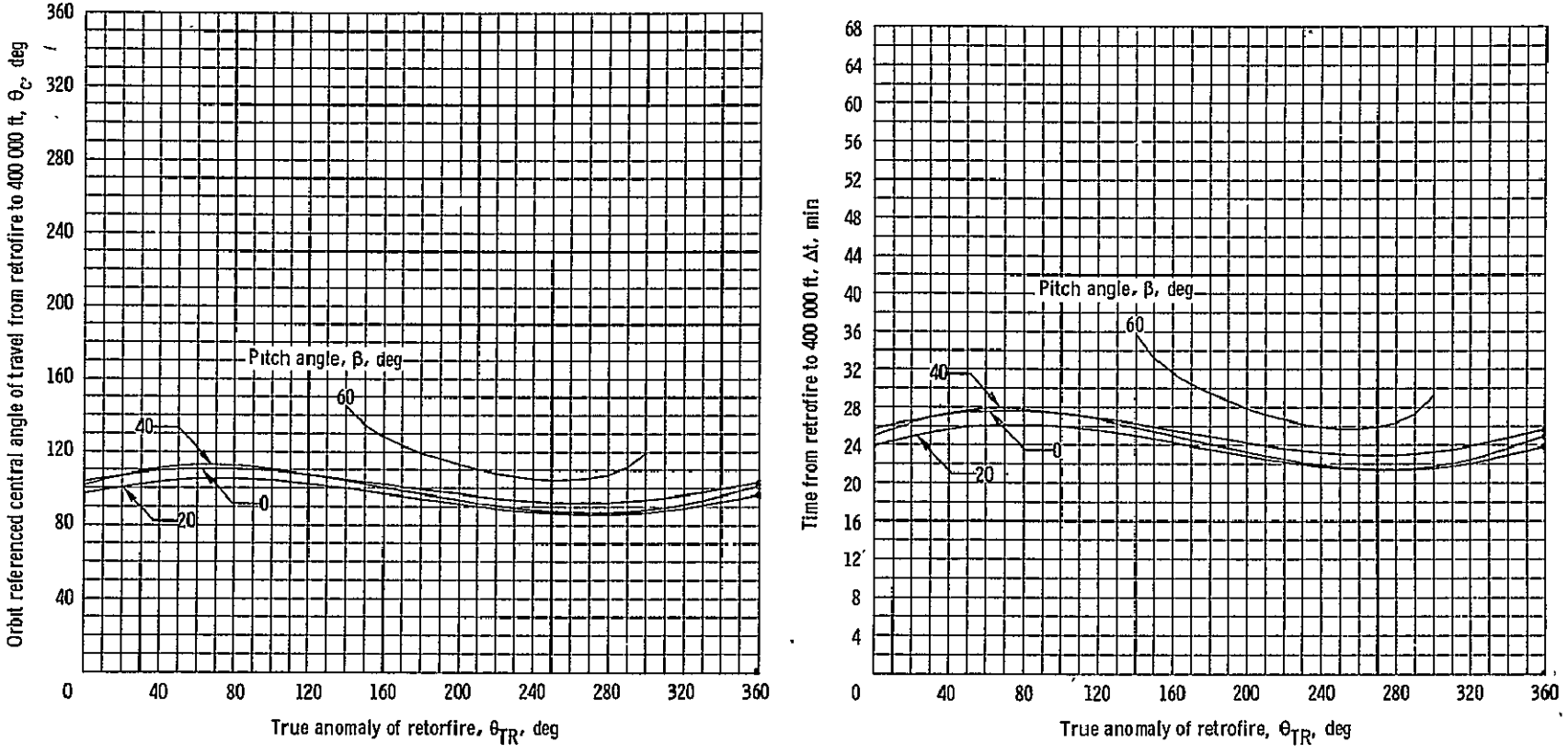
(b) Retrograde ΔV = 500 fps.

Figure 25. – Continued.



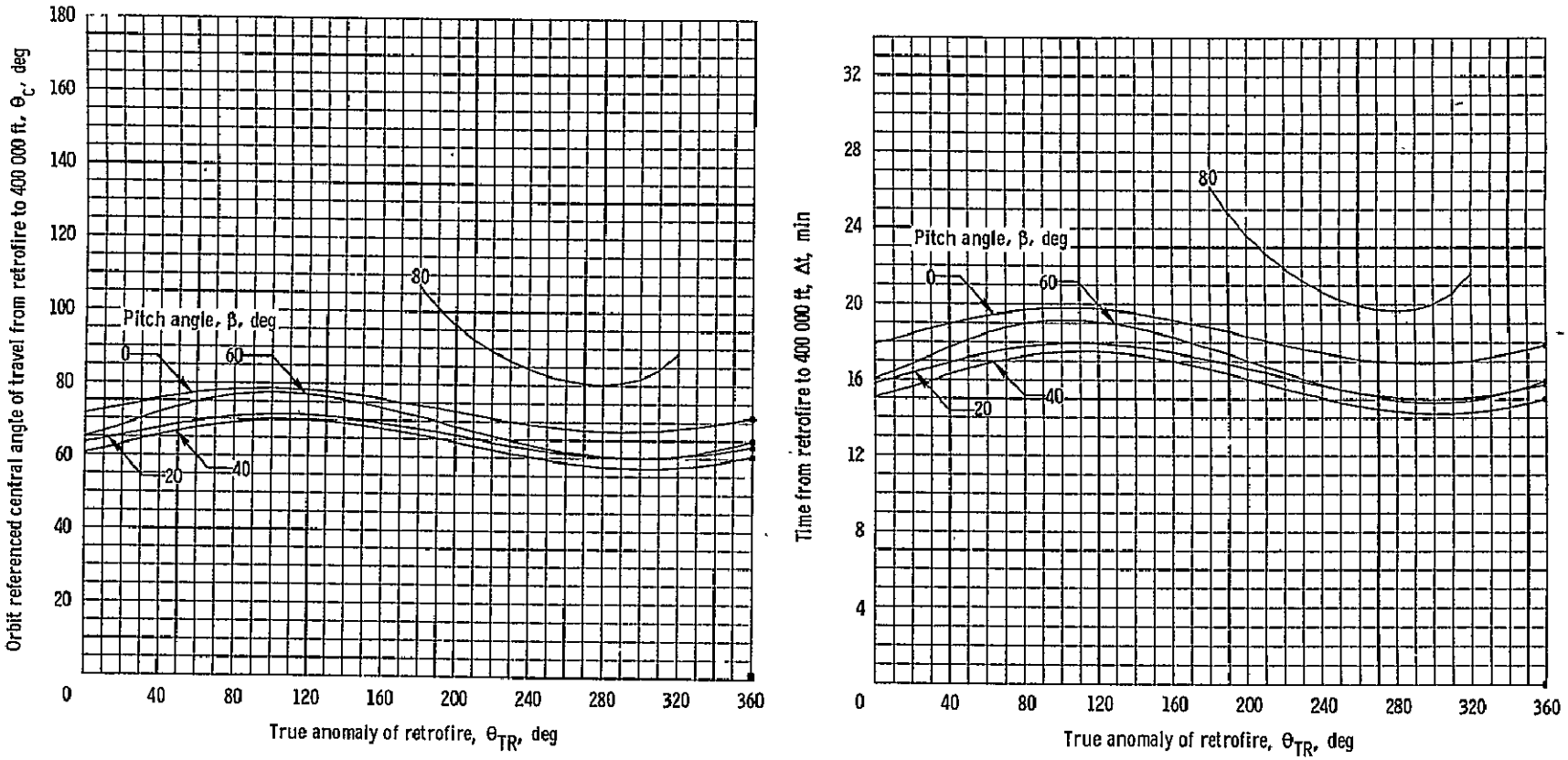
(c) Retrograde ΔV = 700 fps.

Figure 25. - Concluded.



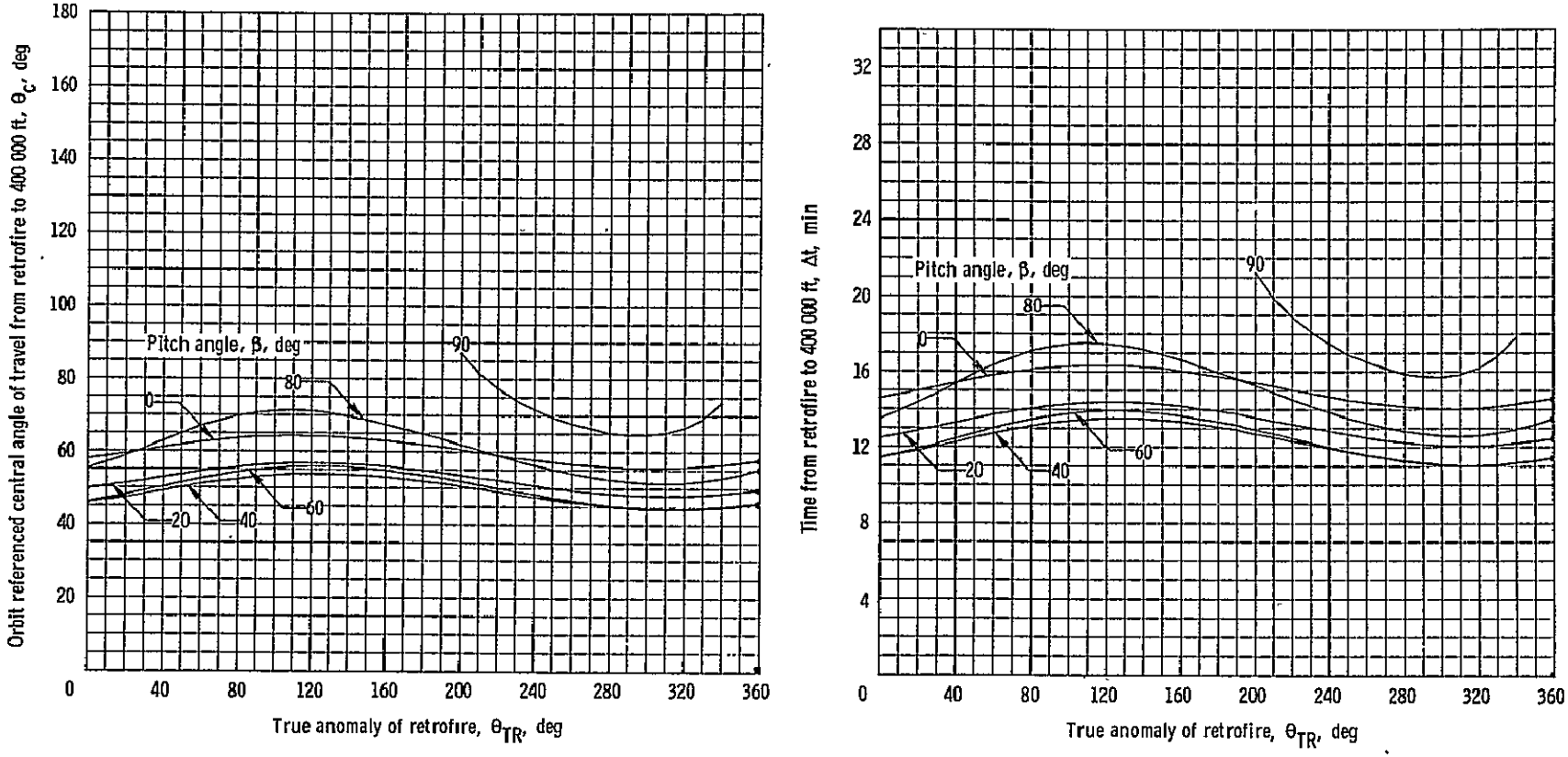
(a) Retrograde ΔV = 300 fps.

Figure 26.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 180 nautical miles and h_p = 153 nautical miles.



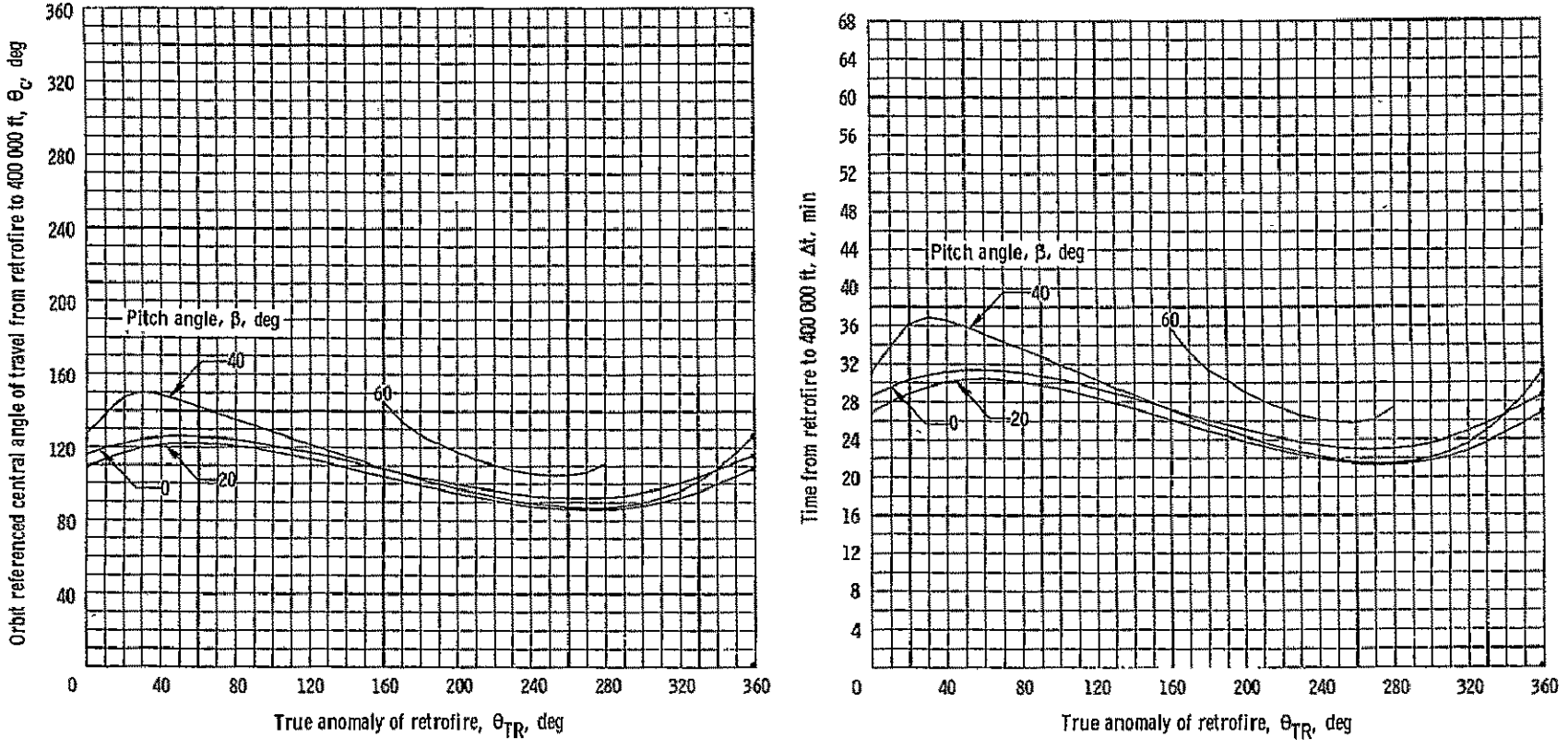
(b) Retrograde ΔV = 500 fps.

Figure 26.- Continued.



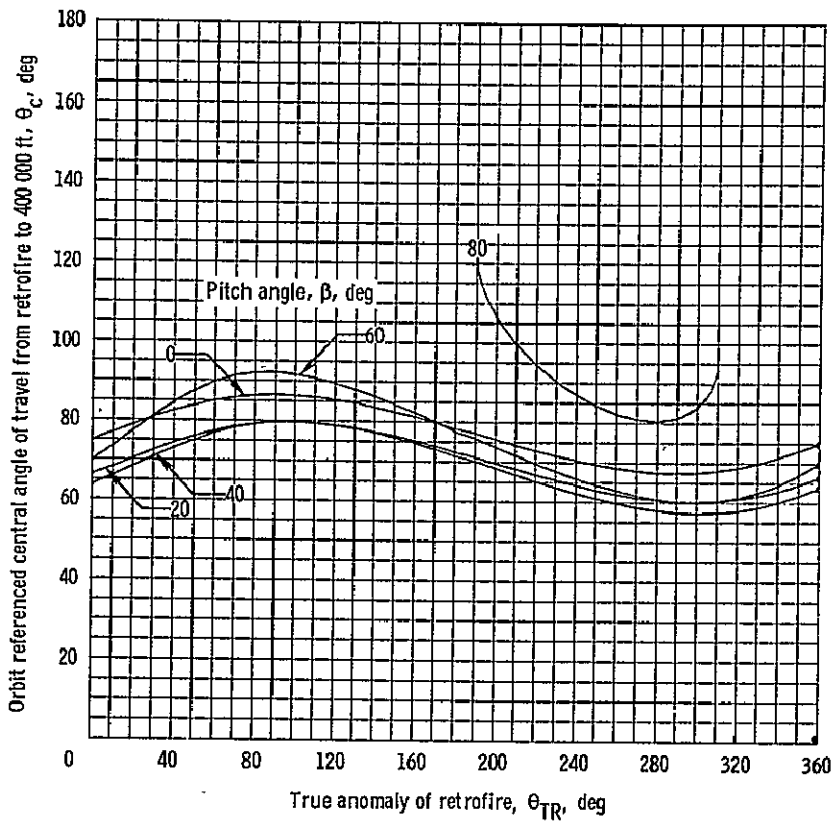
(c) Retrograde ΔV = 700 fps.

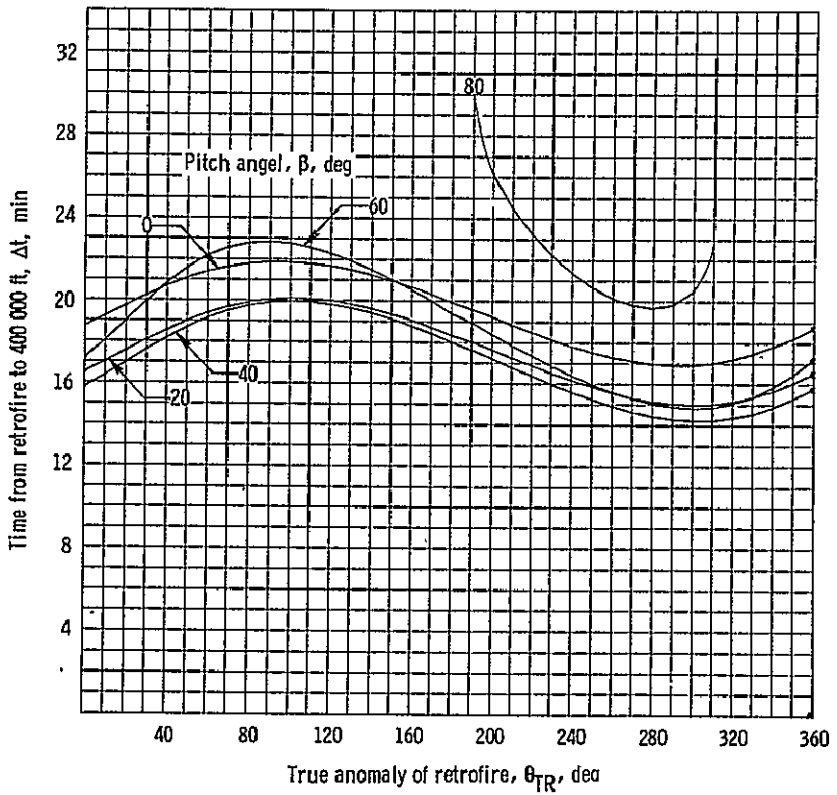
Figure 26. - Concluded.



(a) Retrograde ΔV = 300 fps.

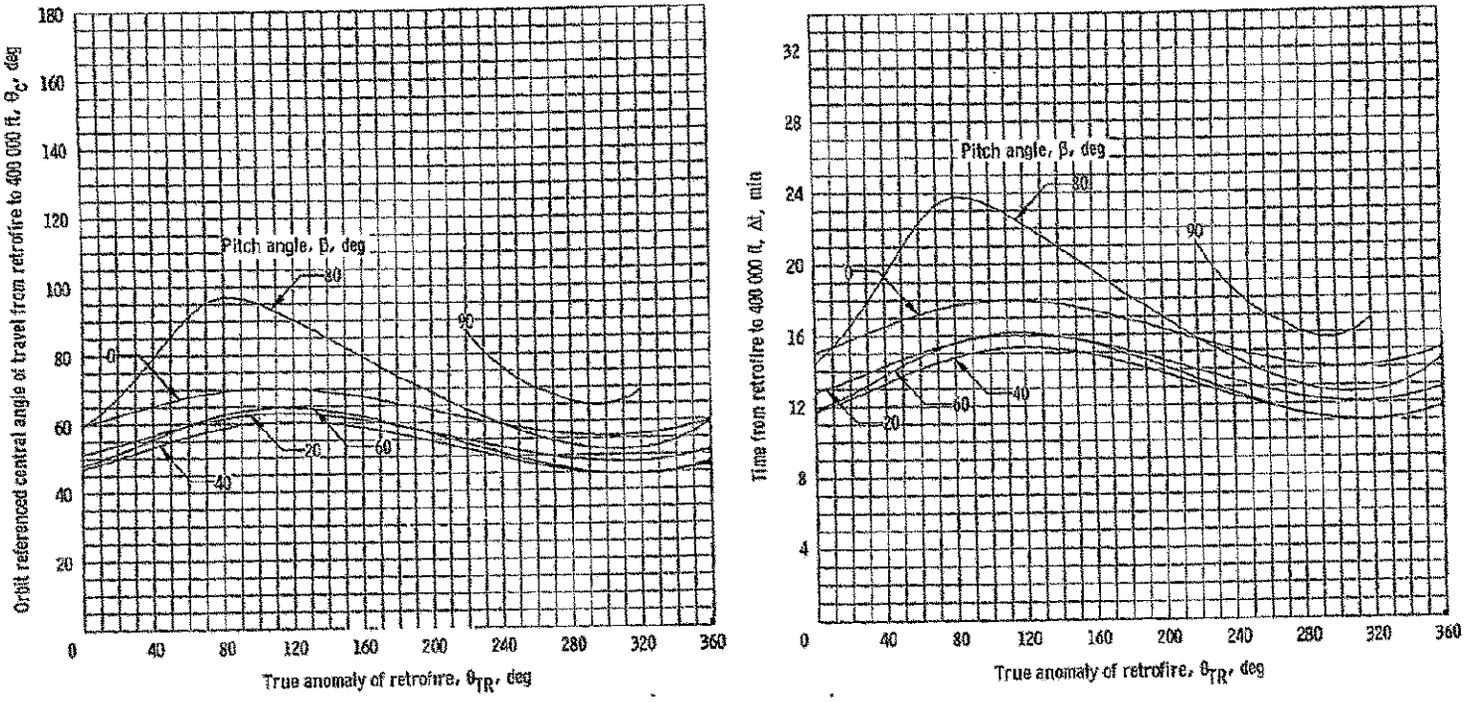
Figure 27.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 200 nautical miles and h_p = 153 nautical miles.





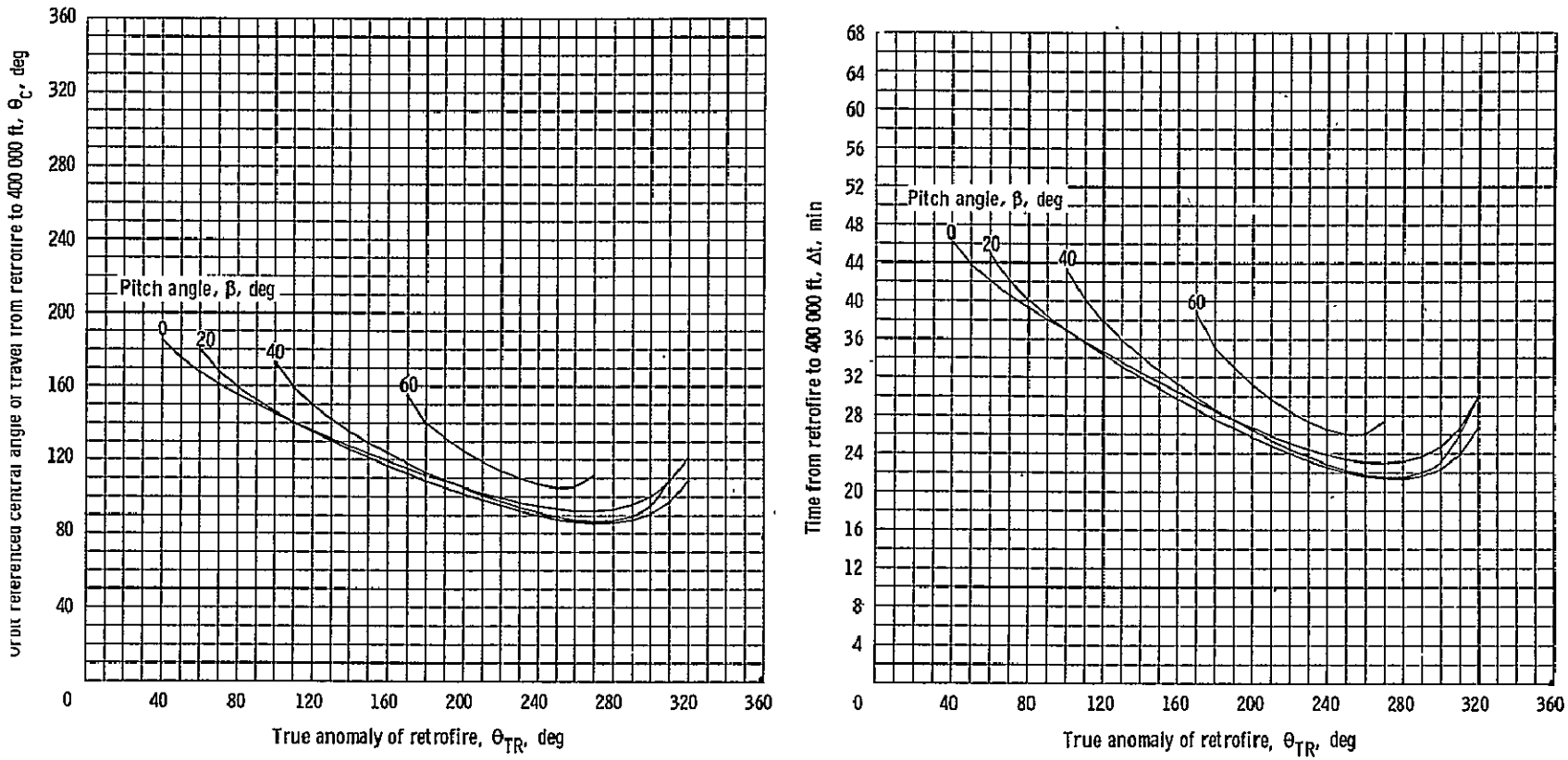
(b) Retrograde ΔV = 500 fps.

Figure 27. - Continued.



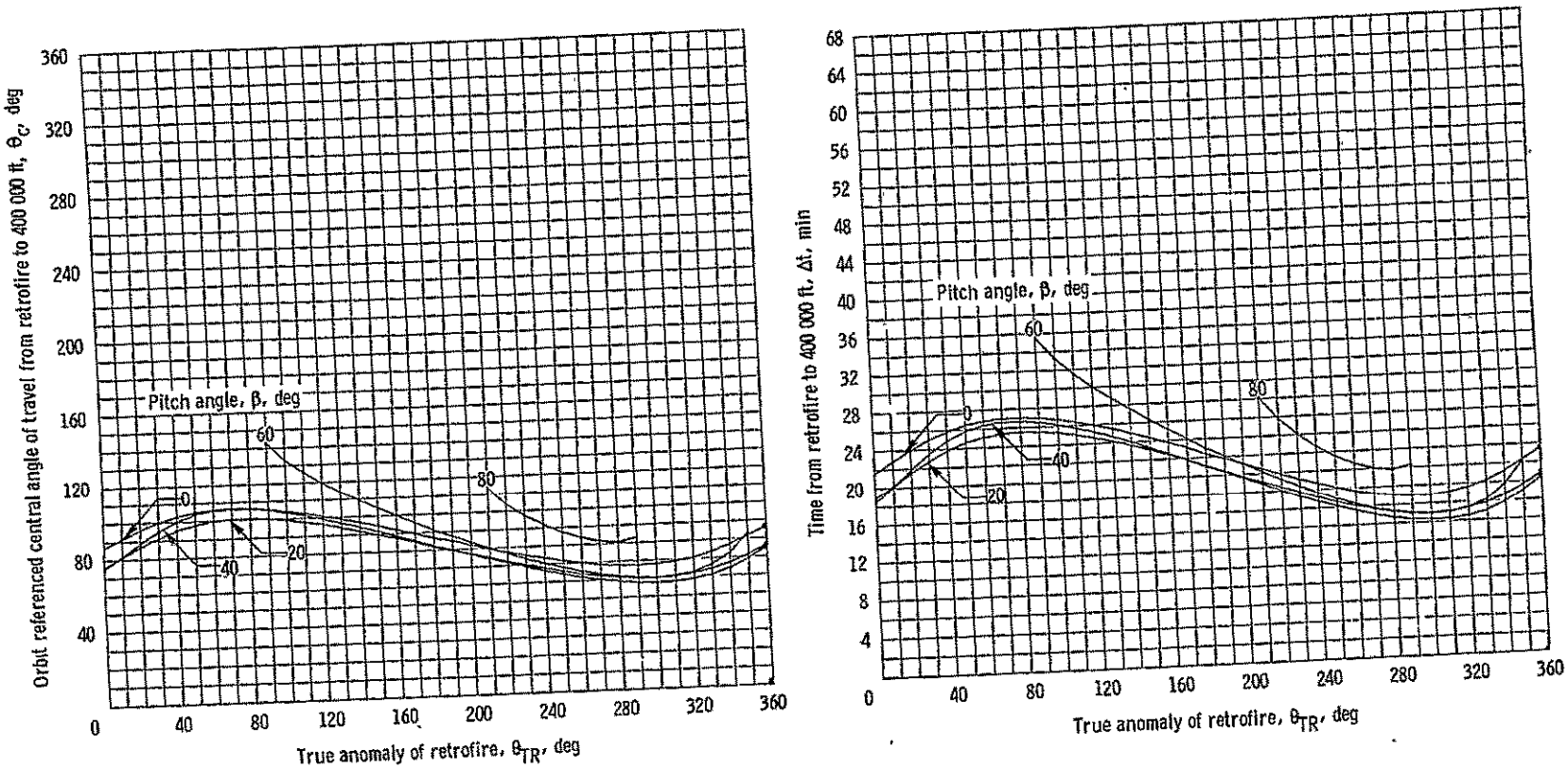
(c) Retrograde ΔV = 700 fps.

Figure 27. - Concluded.



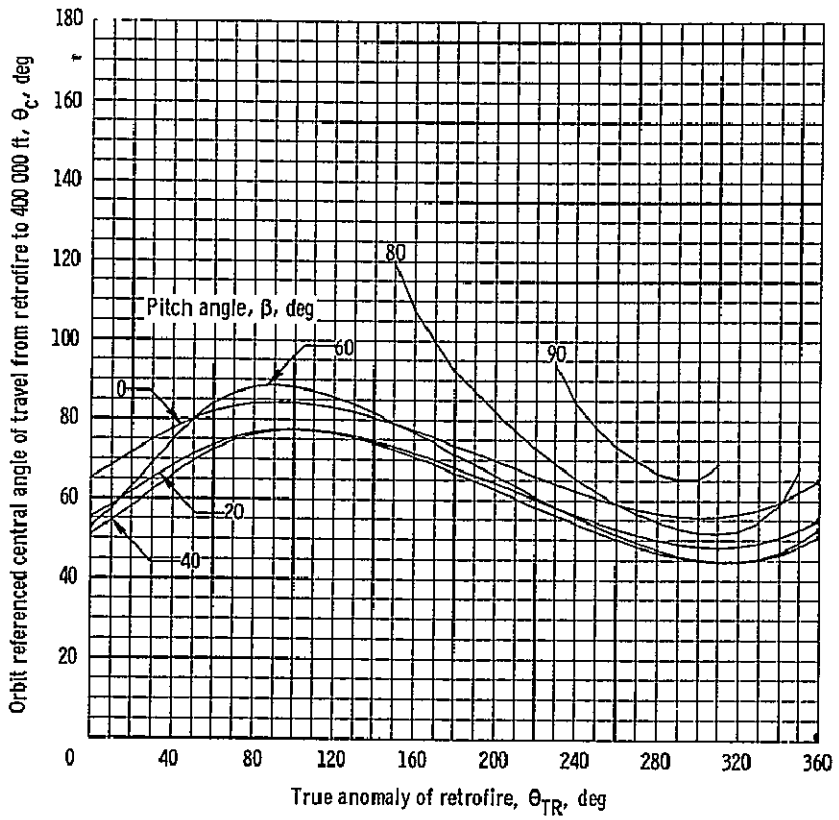
(a) Retrograde ΔV = 300 fps.

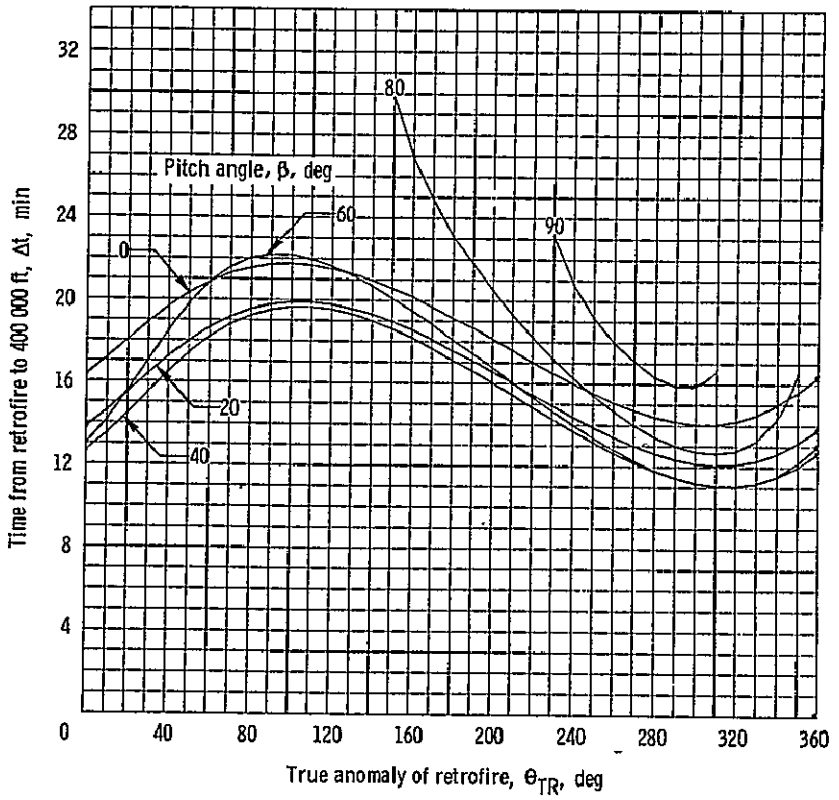
Figure 28.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 250 nautical miles and h_p = 153 nautical miles.



(b) Retrograde ΔV = 500 fps.

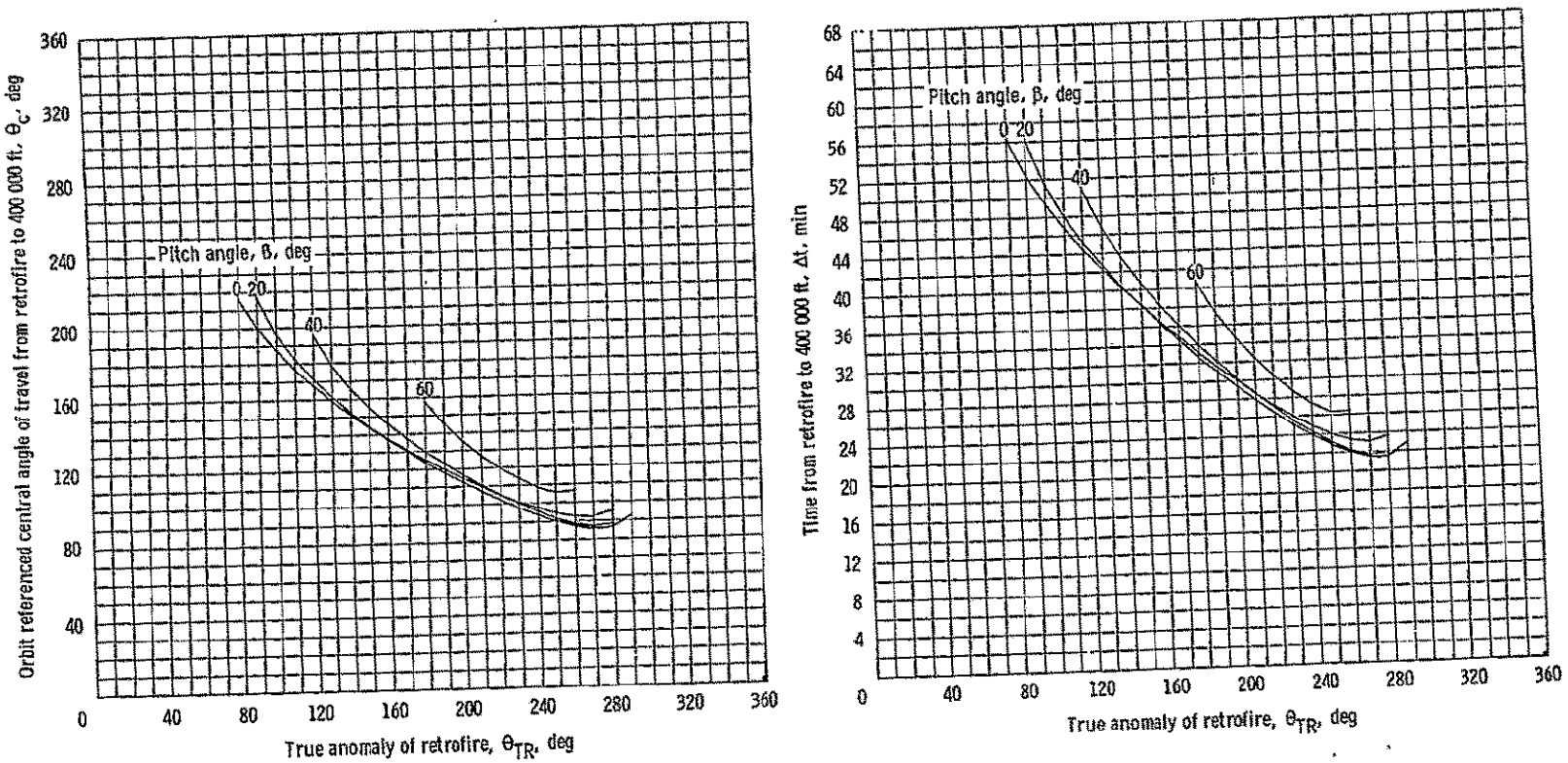
Figure 28.- Continued.





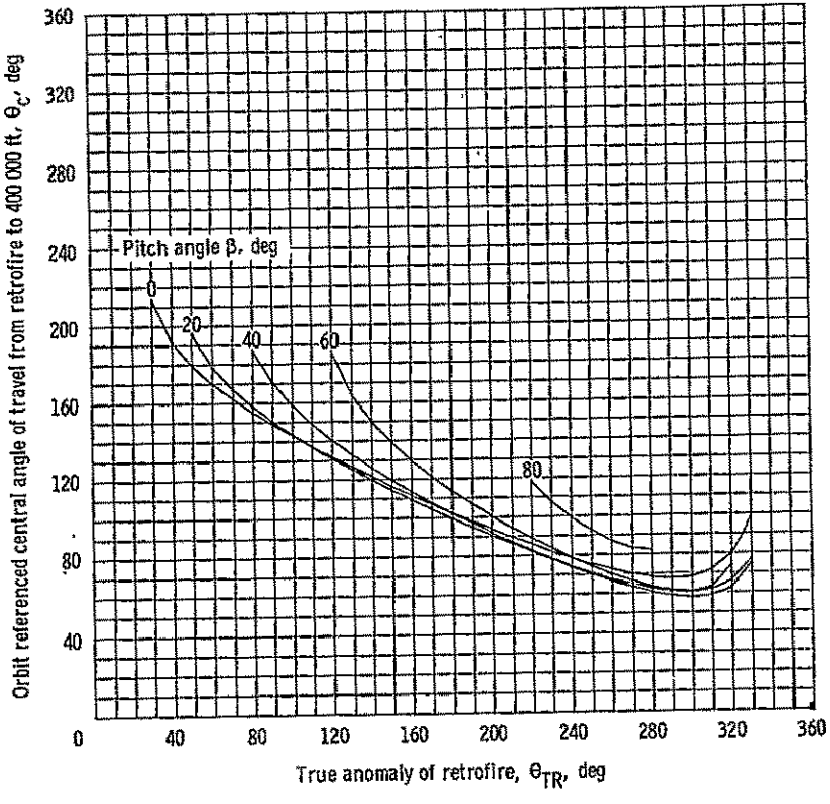
(c) Retrograde ΔV = 700 fps.

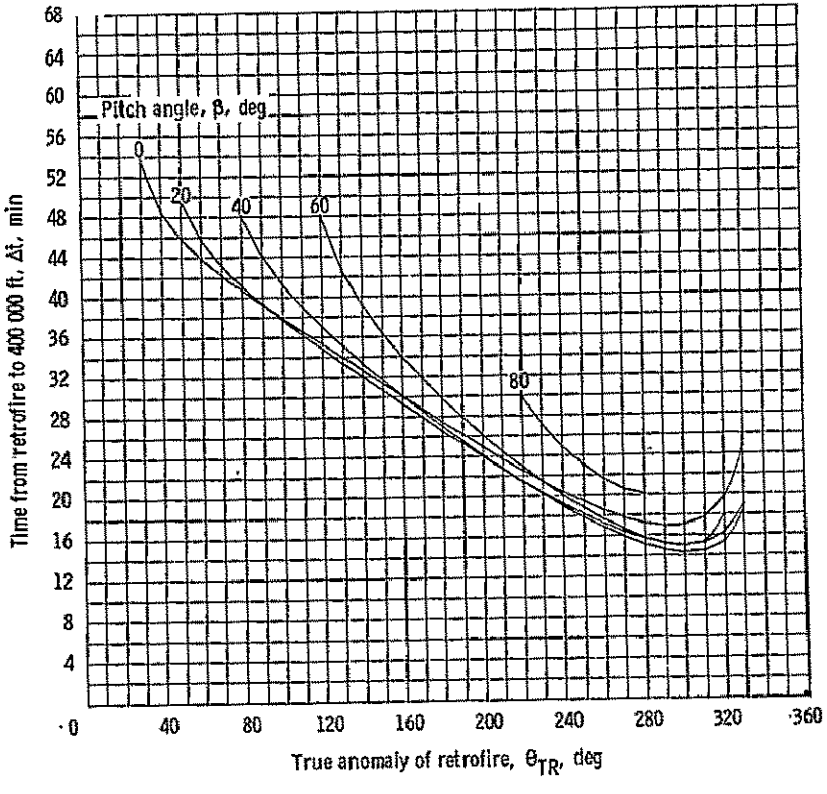
Figure 28.- Concluded.



(a) Retrograde ΔV = 300 fps.

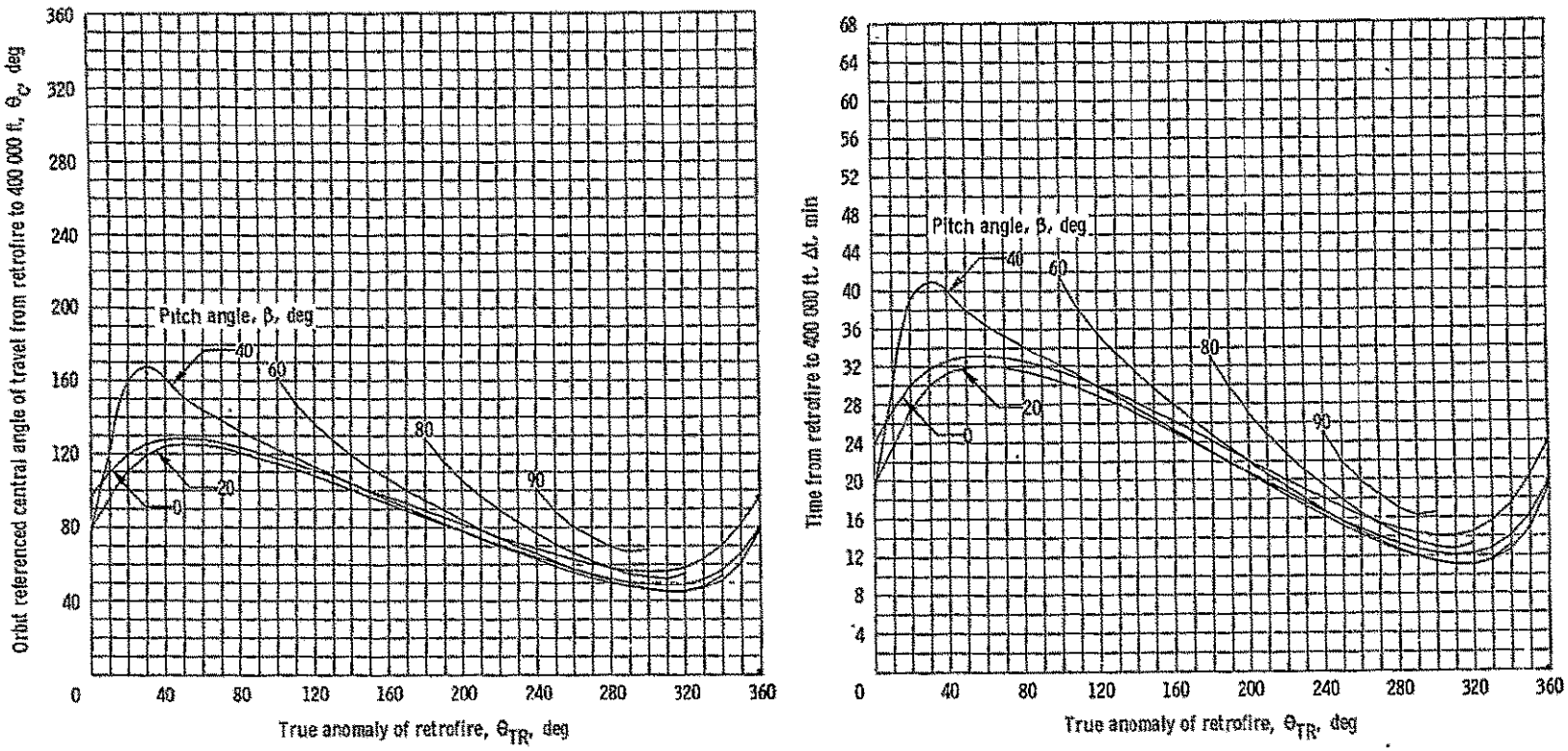
Figure 29. – Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 400 nautical miles and h_p = 153 nautical miles.





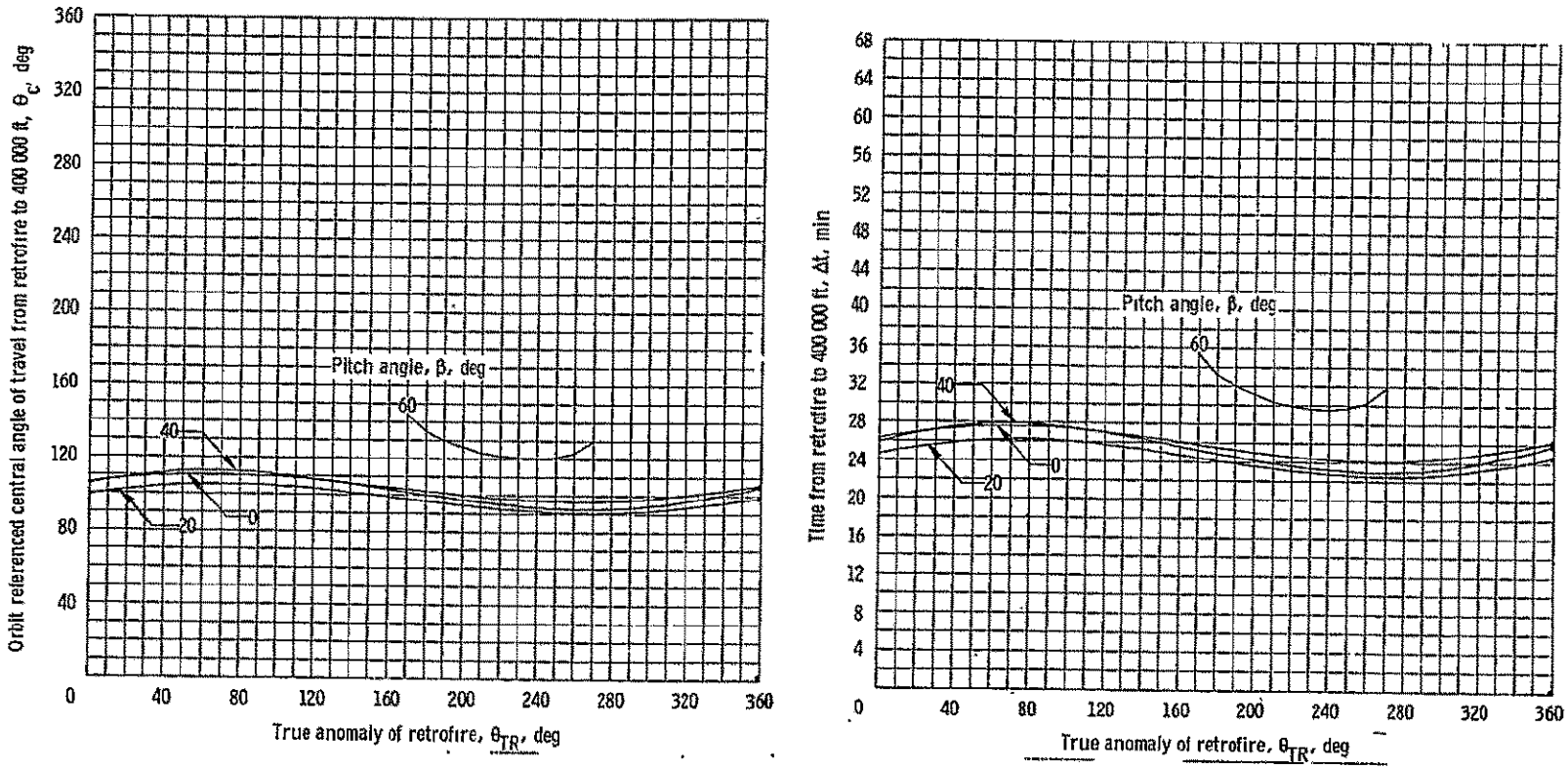
(b) Retrograde ΔV = 500 fps.

Figure 29.- Continued.



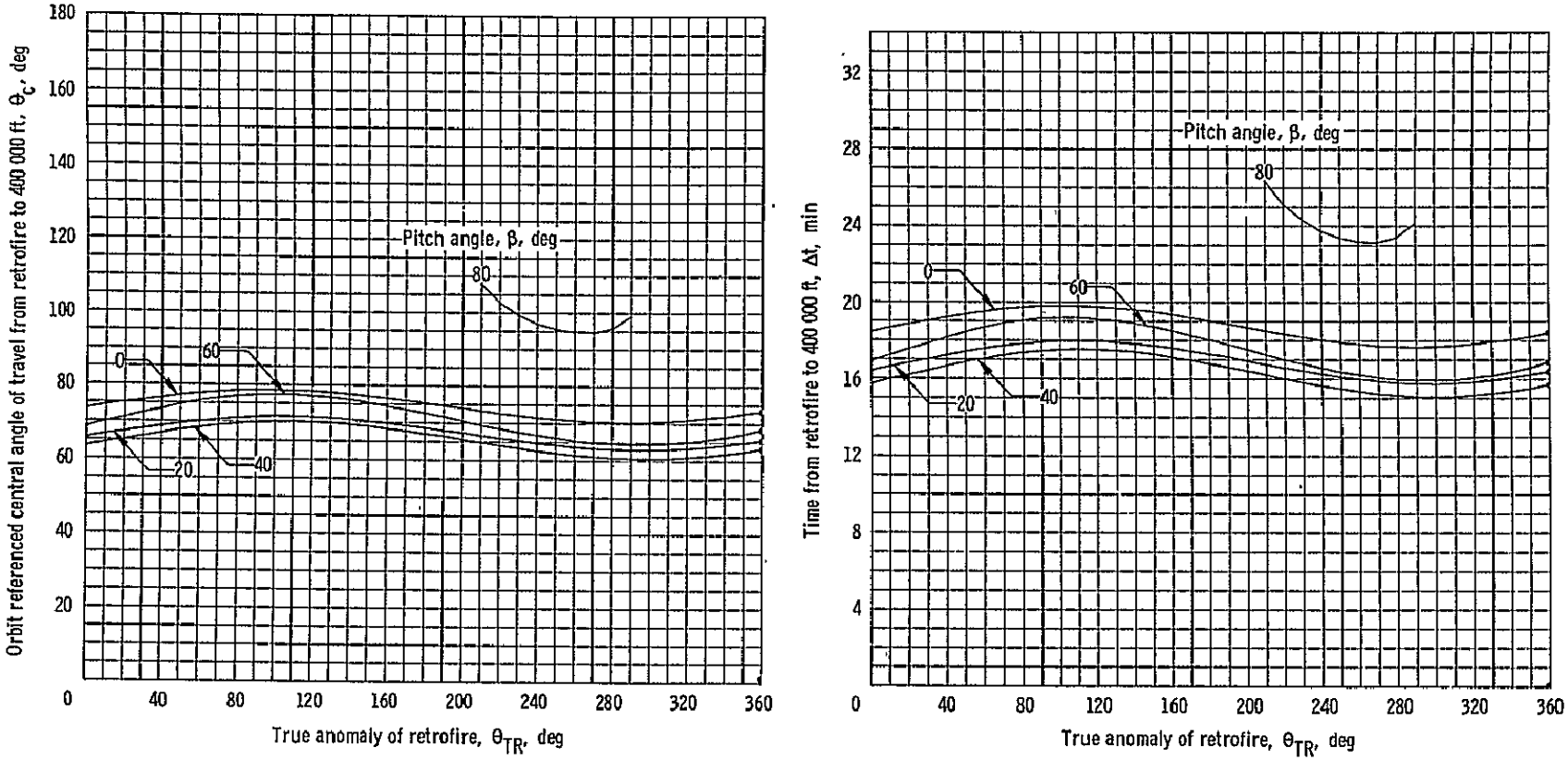
(c) Retrograde ΔV = 700 fps.

Figure 29.- Concluded.



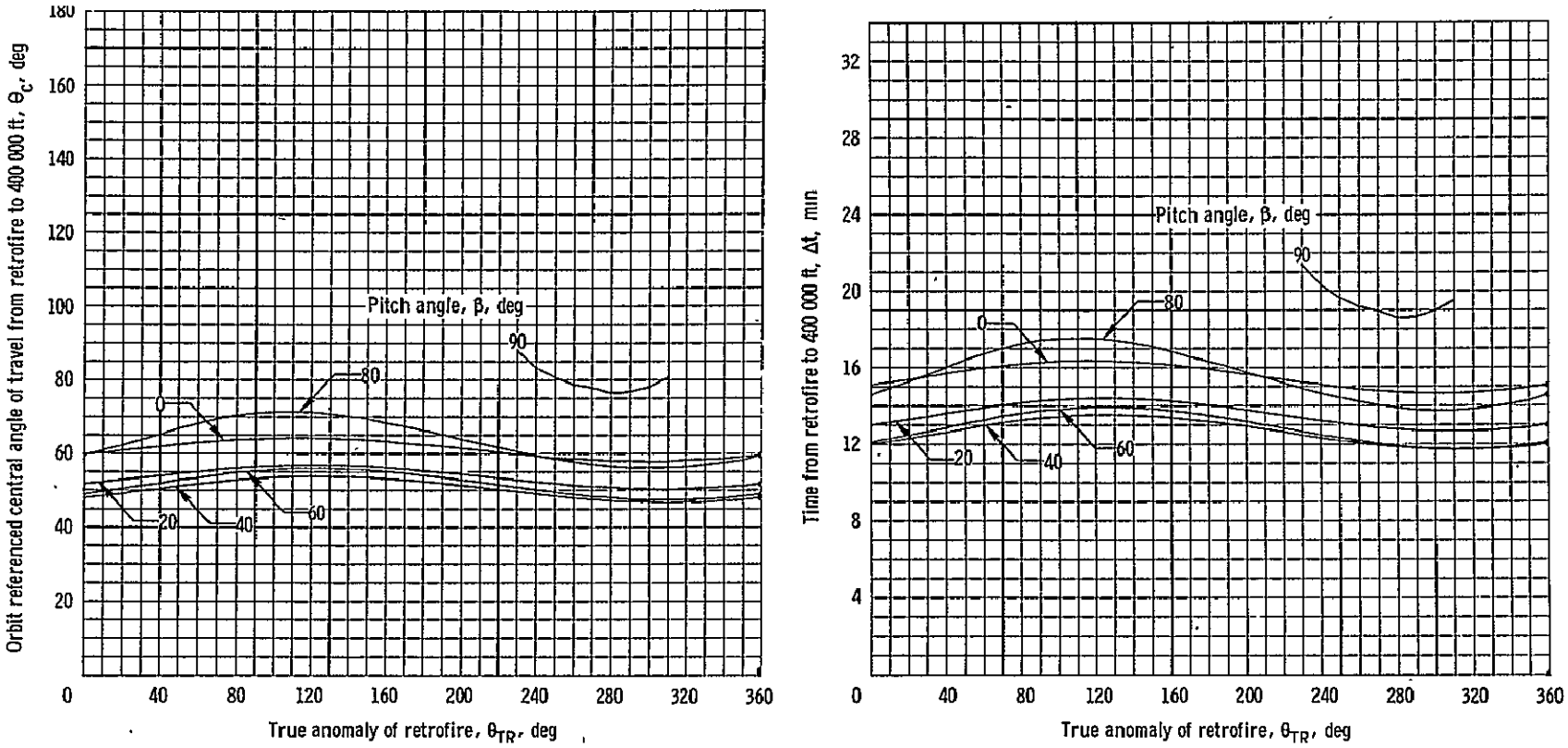
(a) Retrograde ΔV = 300 fps.

Figure 30. - Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 180 nautical miles and h_p = 160 nautical miles.



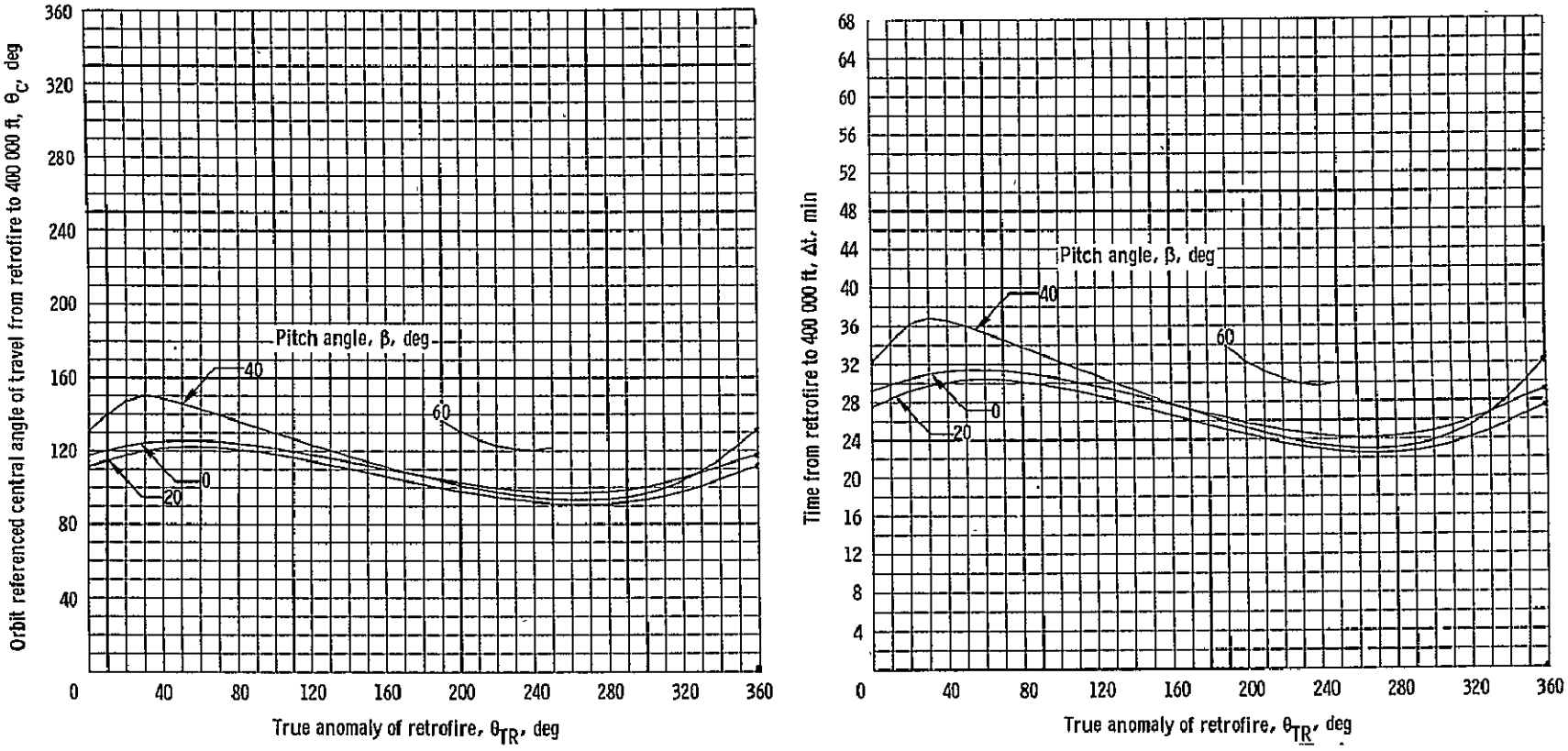
(b) Retrograde ΔV = 500 fps.

Figure 30.- Continued.



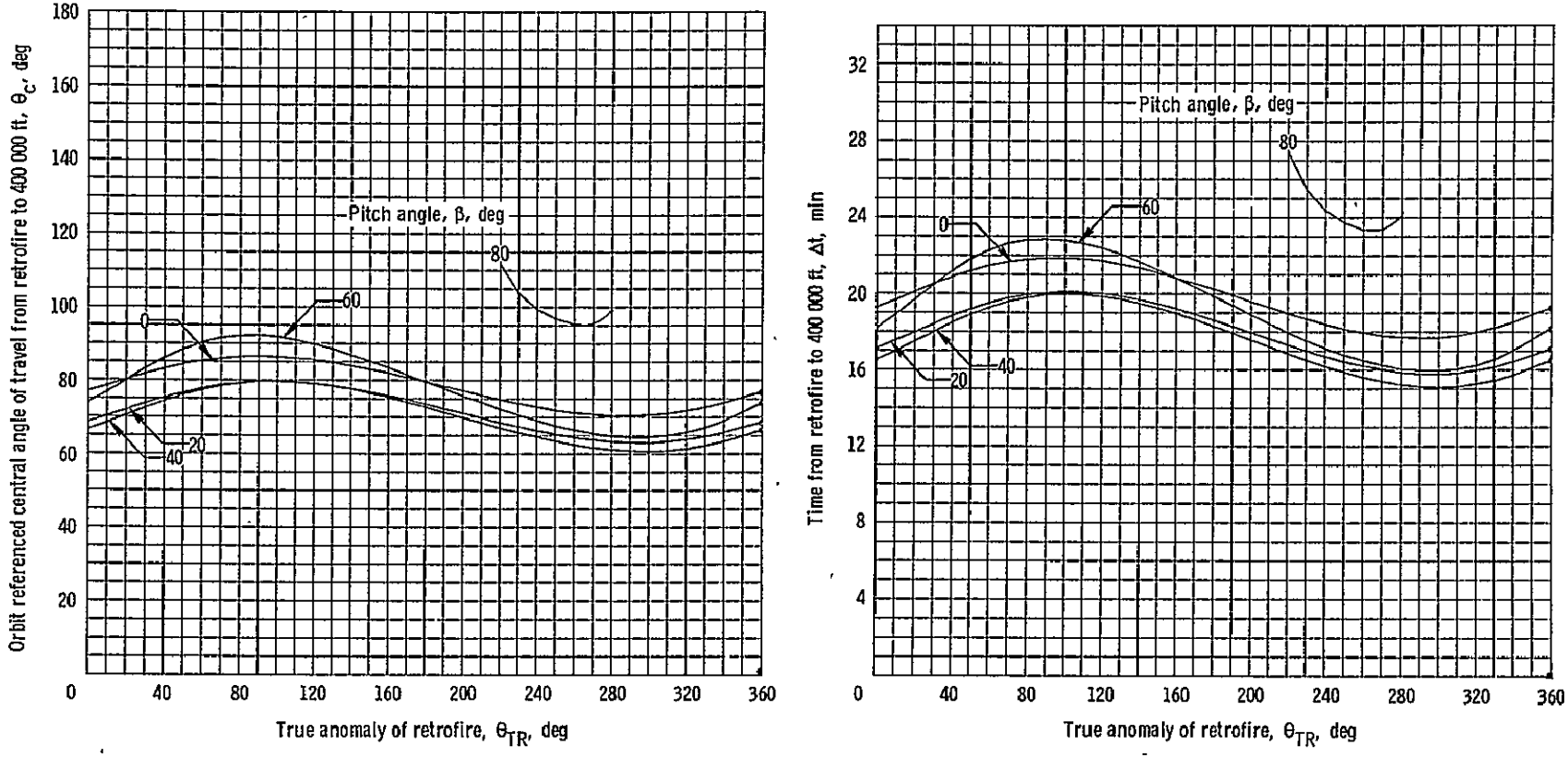
(c) Retrograde ΔV = 700 fps.

Figure 30. - Concluded.



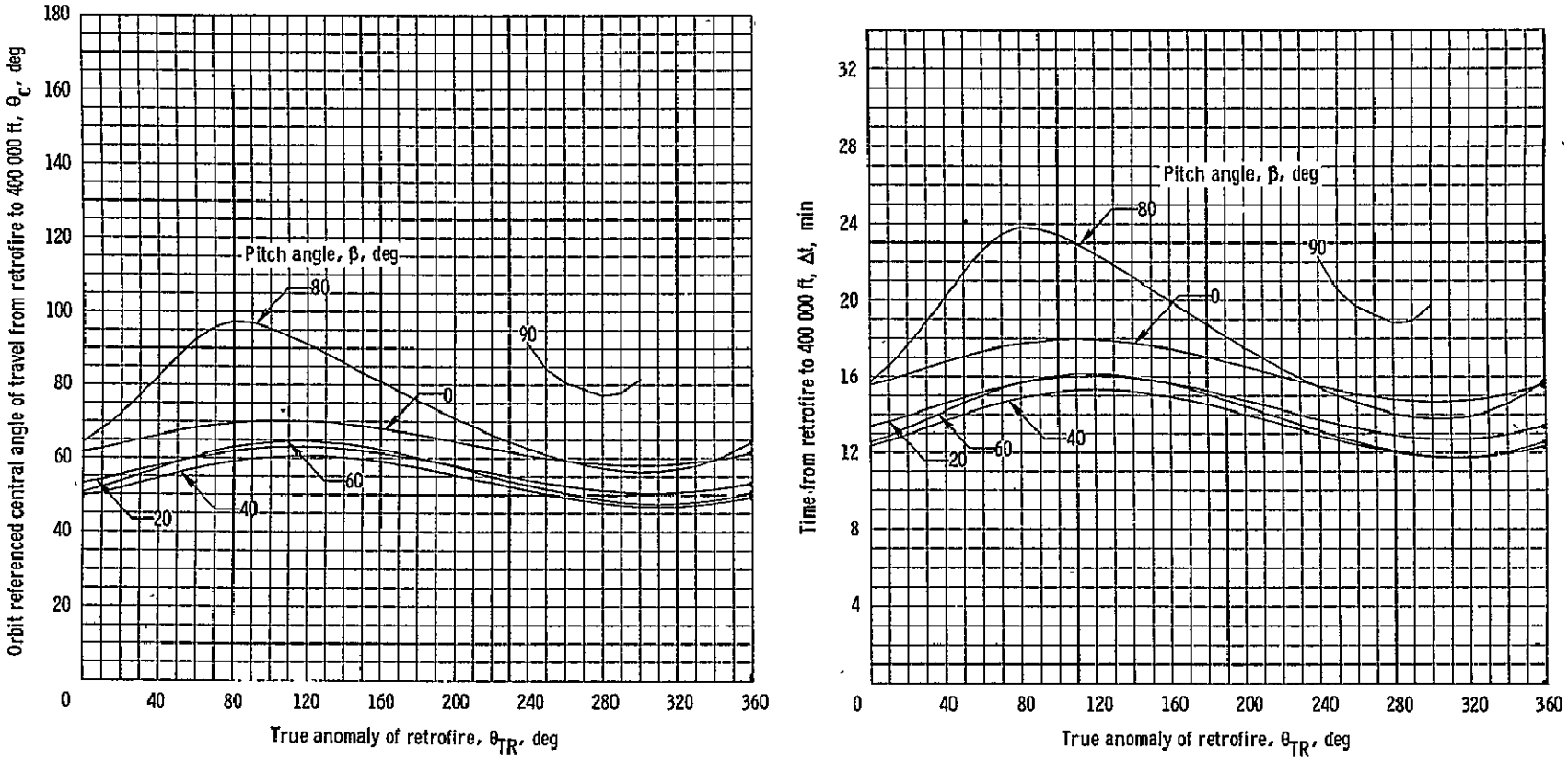
(a) Retrograde ΔV = 300 fps.

Figure 31. - Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 200 nautical miles and h_p = 160 nautical miles.



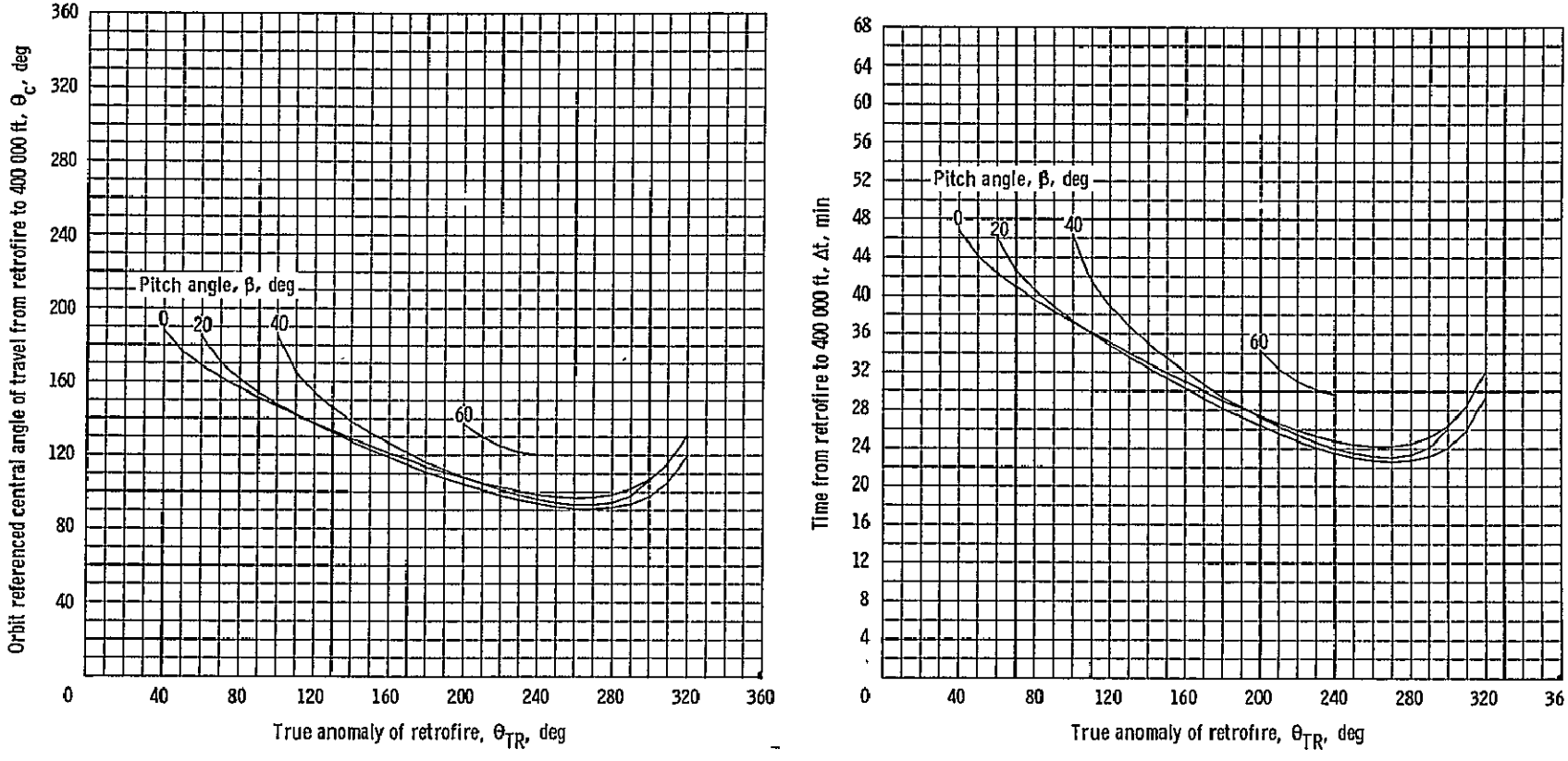
(b) Retrograde ΔV = 500 fps.

Figure 31. - Continued.



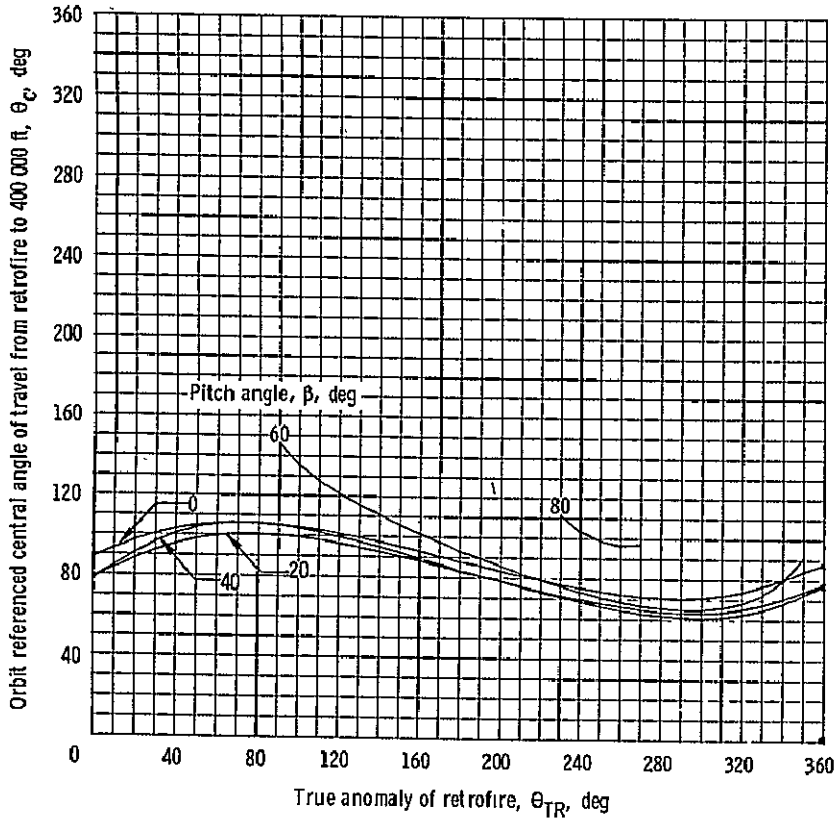
(c) Retrograde ΔV = 700 fps.

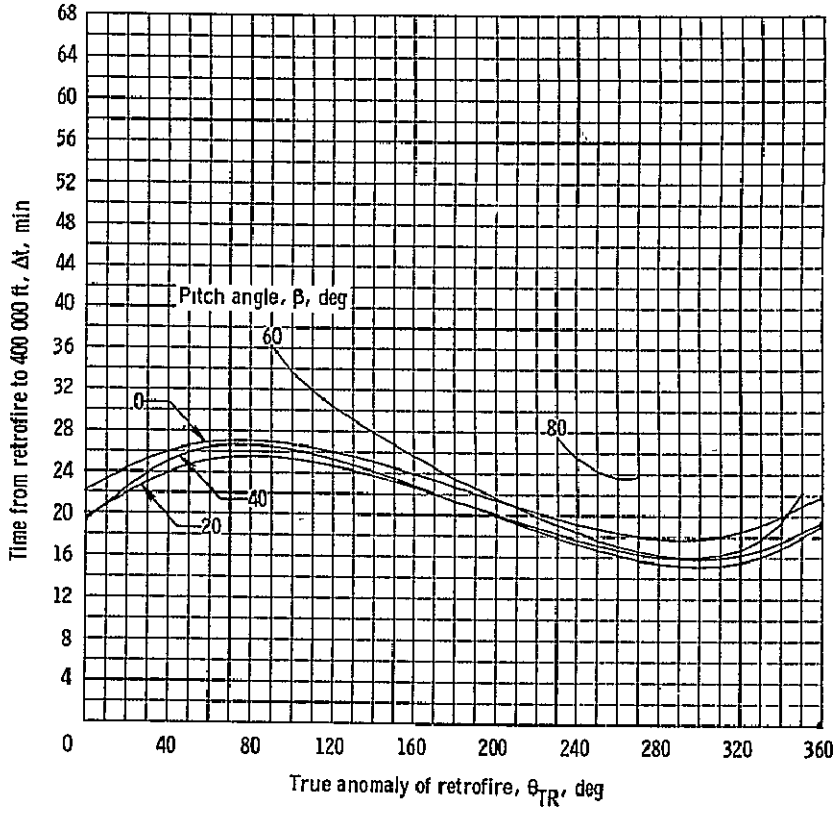
Figure 31. - Concluded.



(a) Retrograde ΔV = 300 fps.

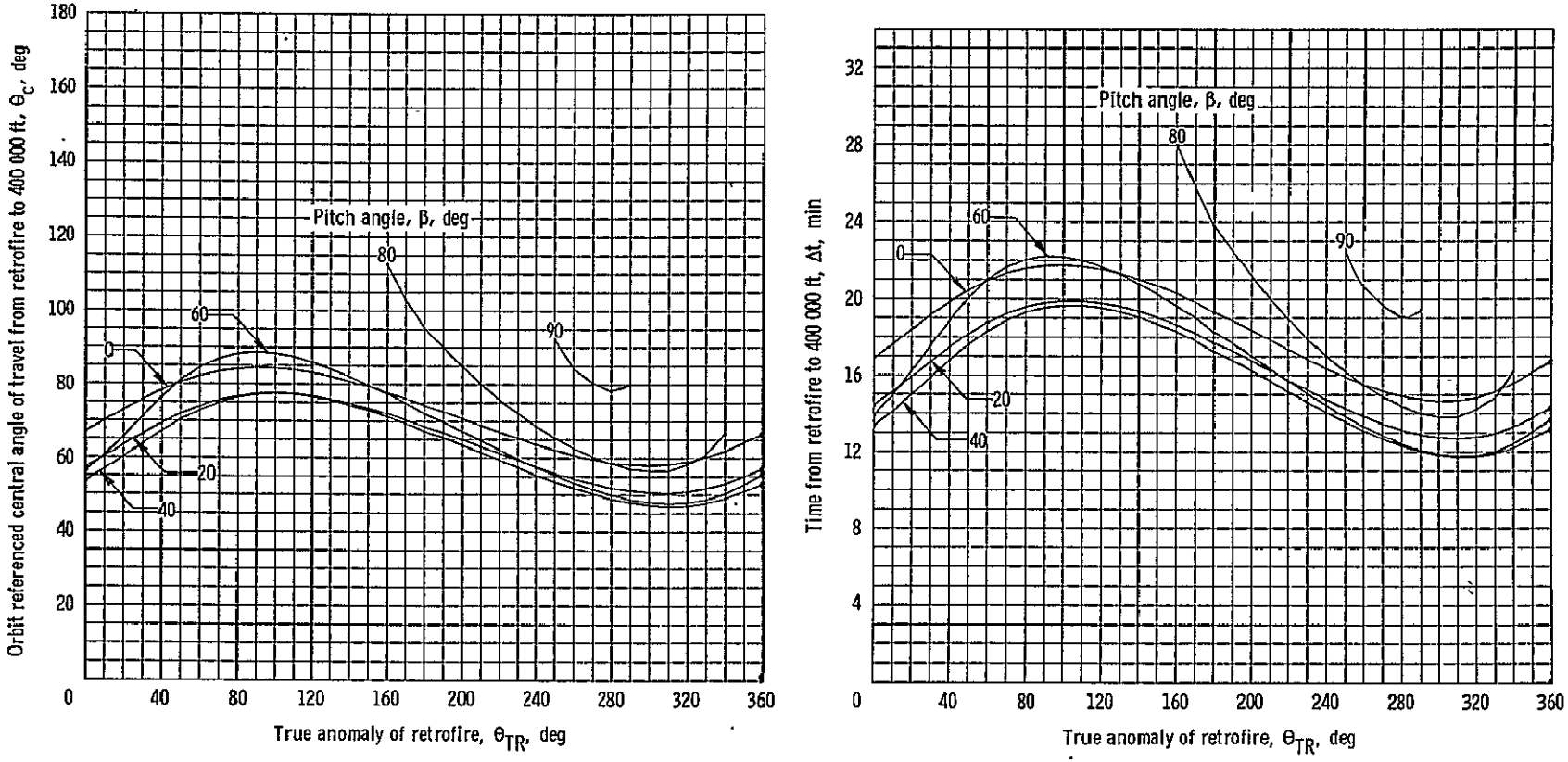
Figure 32. – Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 250 nautical miles and h_p = 160 nautical miles.





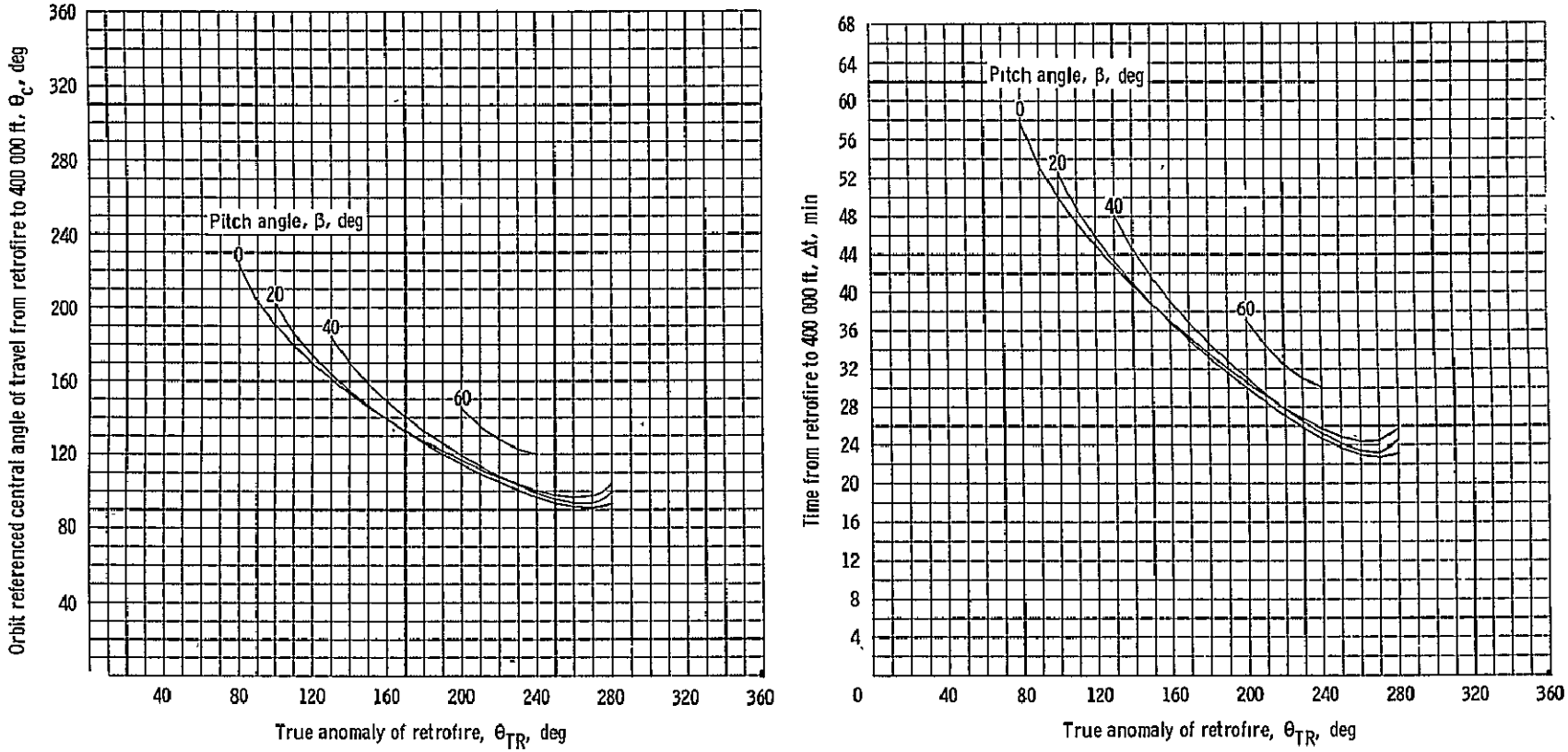
(b) Retrograde ΔV = 500 fps.

Figure 32. - Continued.



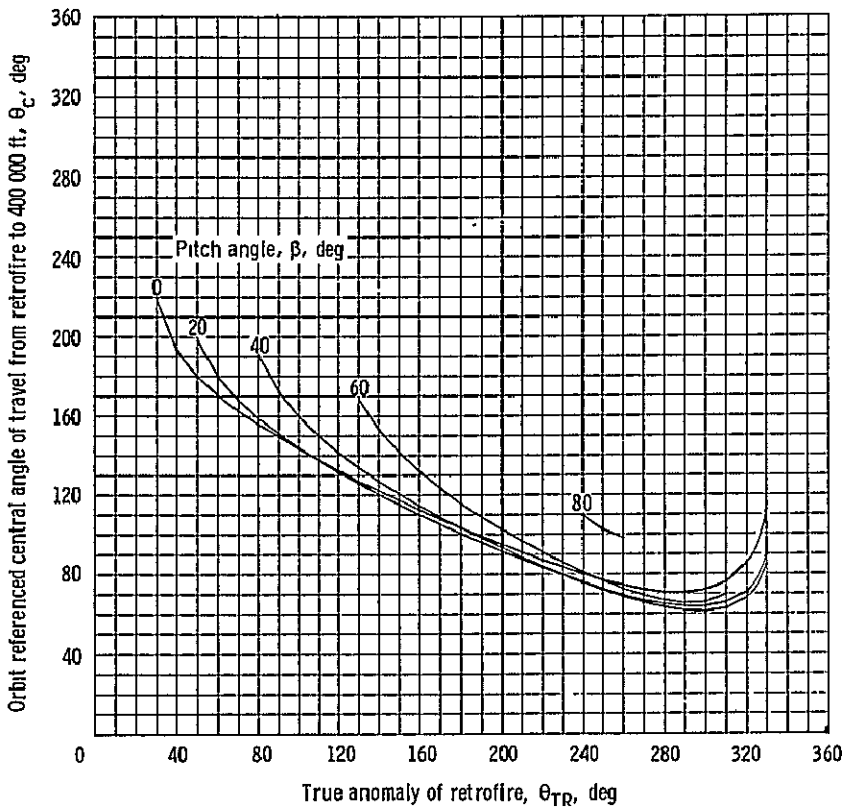
(c) Retrograde ΔV = 700 fps.

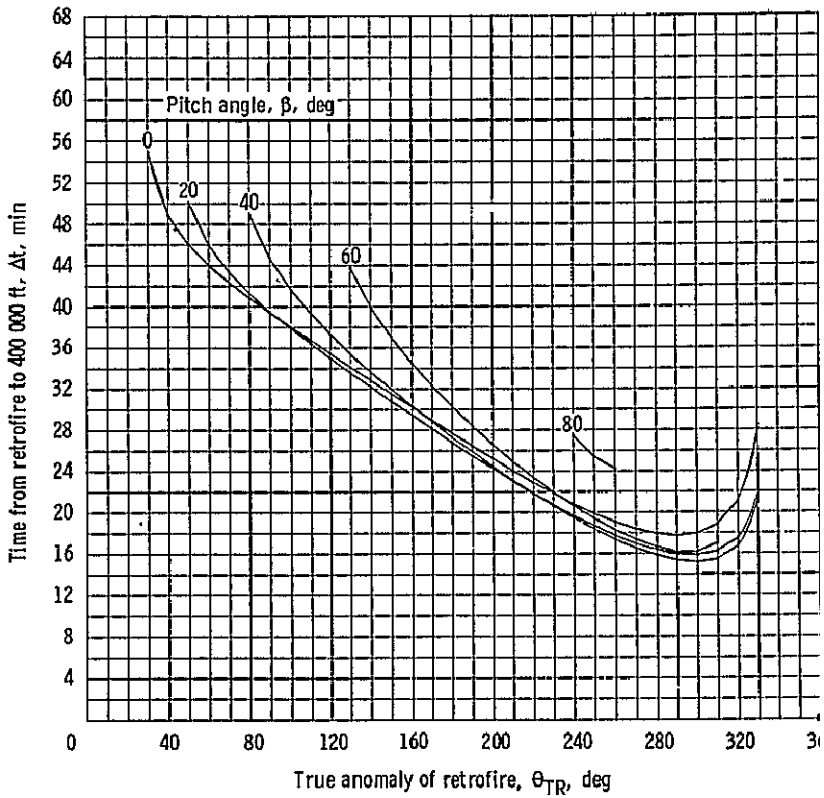
Figure 32. - Concluded.



(a) Retrograde ΔV = 300 fps.

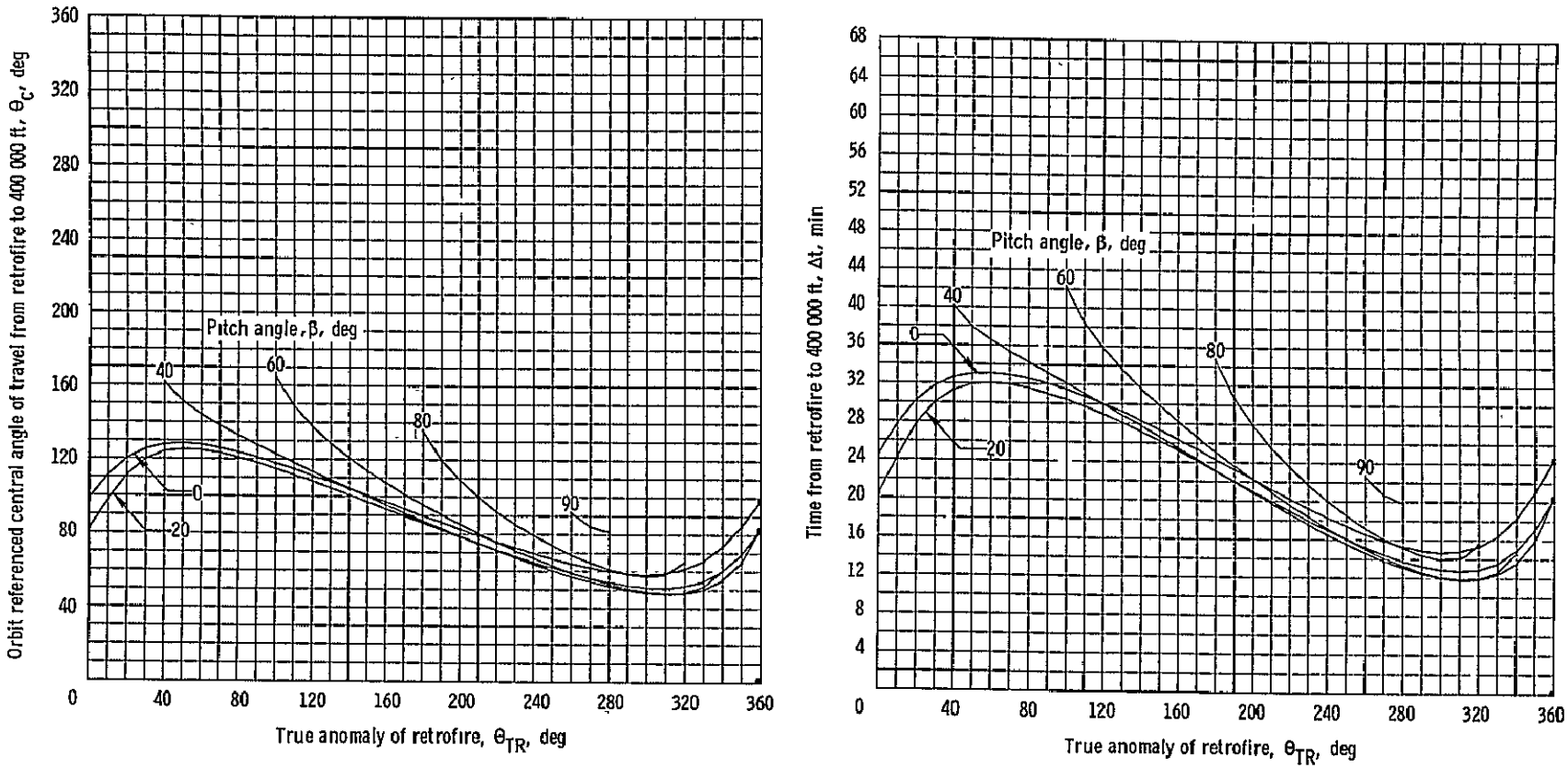
Figure 33. – Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 400 nautical miles and h_p = 160 nautical miles.





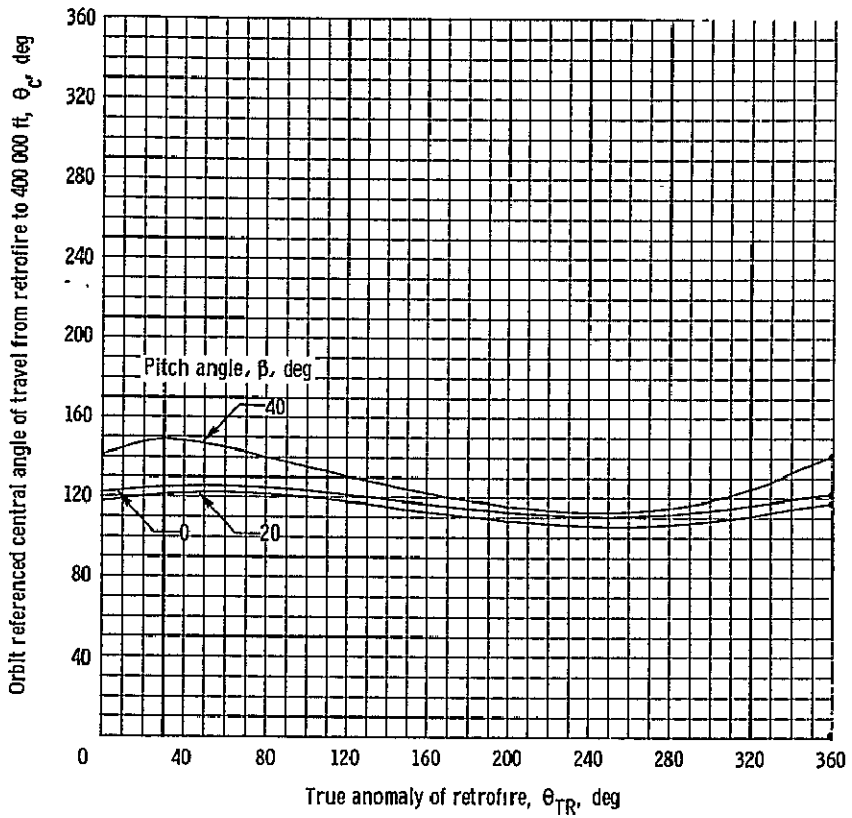
(b) Retrograde ΔV = 500 fps.

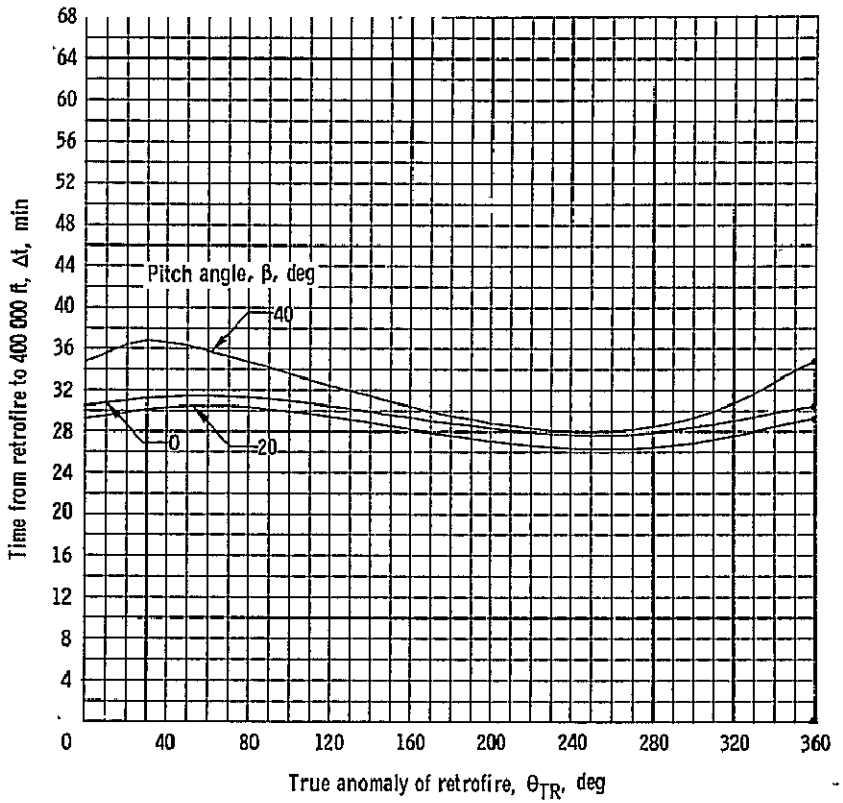
Figure 33.- Continued.



(c) Retrograde ΔV = 700 fps.

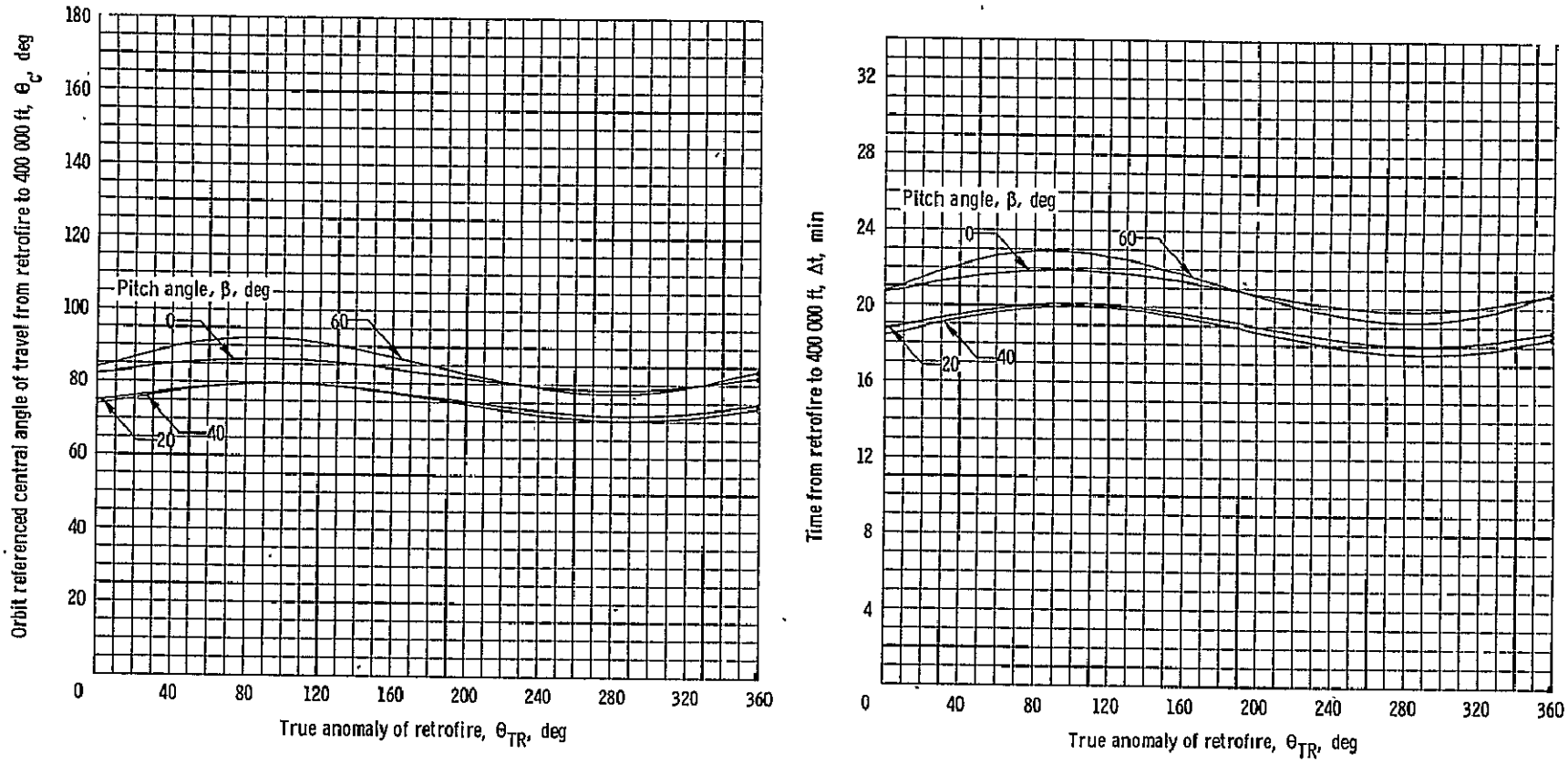
Figure 33. - Concluded.





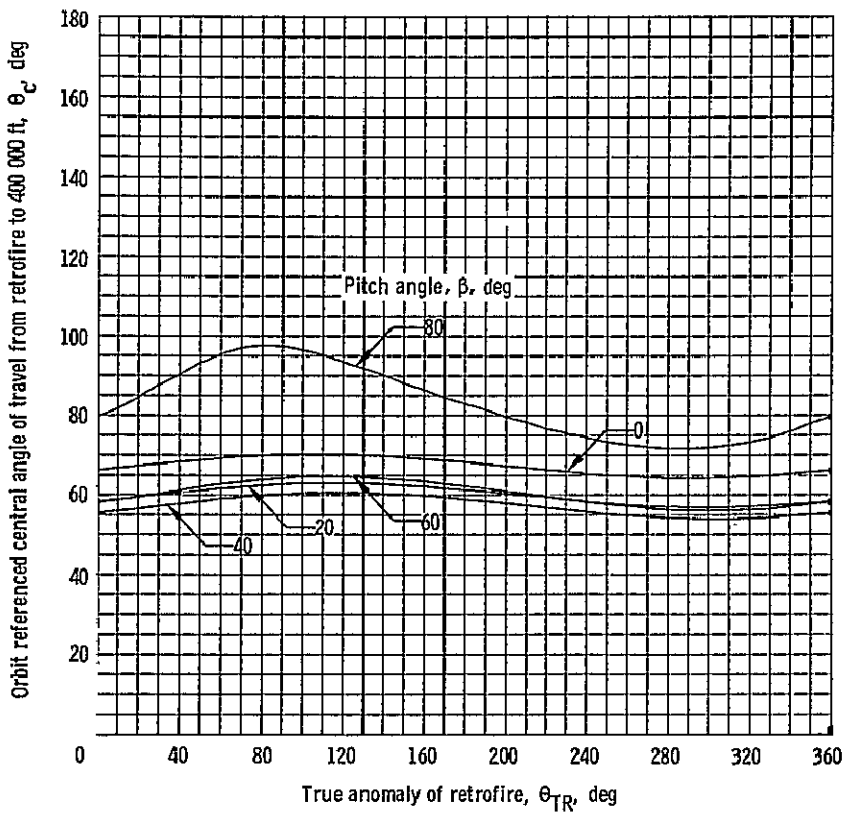
(a) Retrograde ΔV = 300 fps.

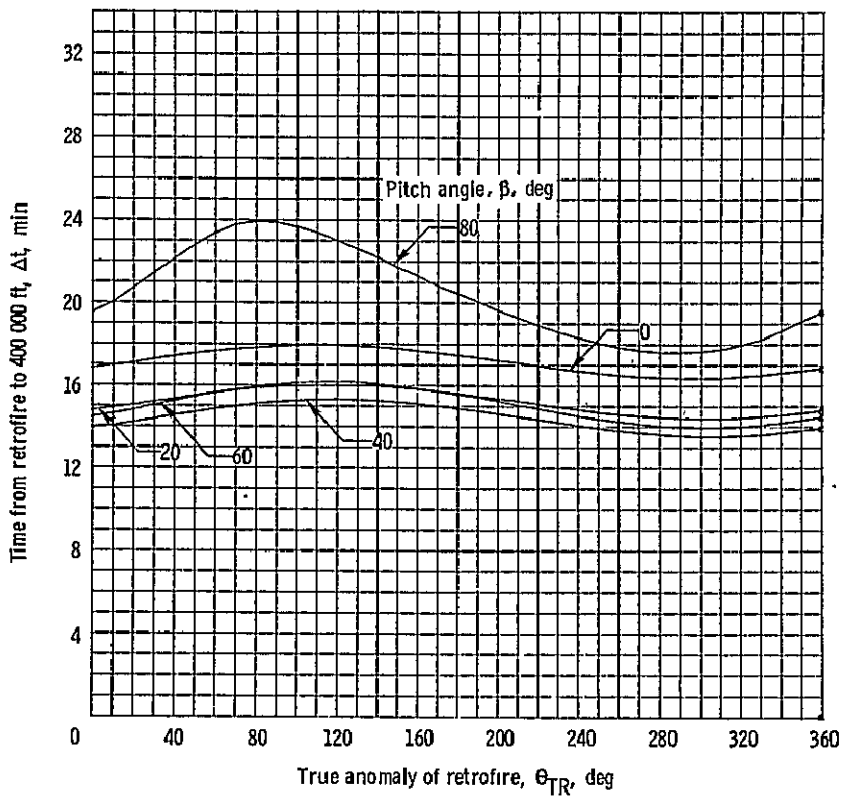
Figure 34.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 200 nautical miles and h_p = 180 nautical miles.



(b) Retrograde ΔV = 500 fps.

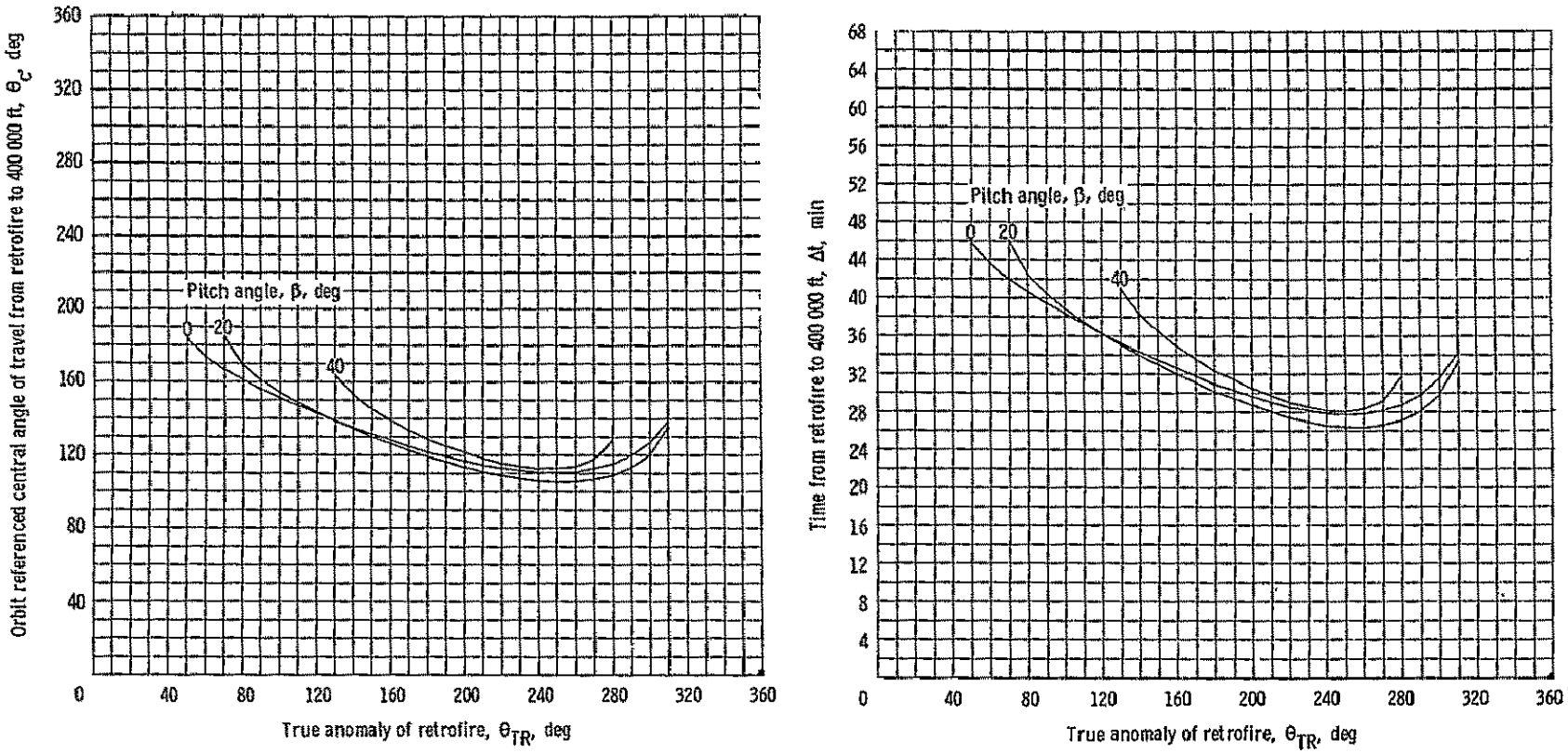
Figure 34. - Continued.





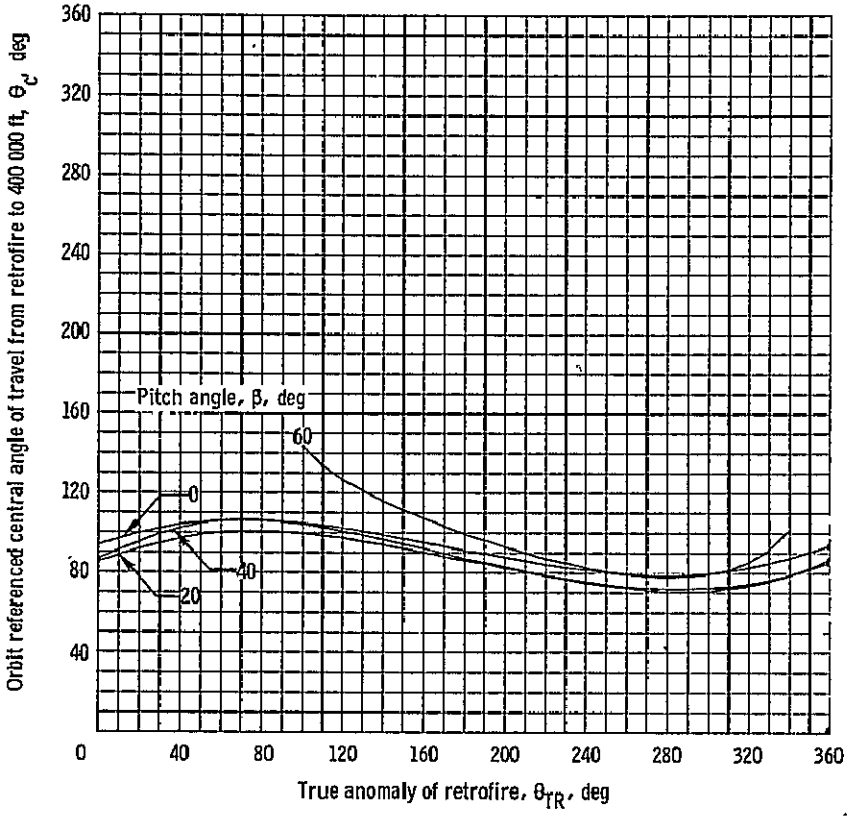
(c) Retrograde ΔV = 700 fps.

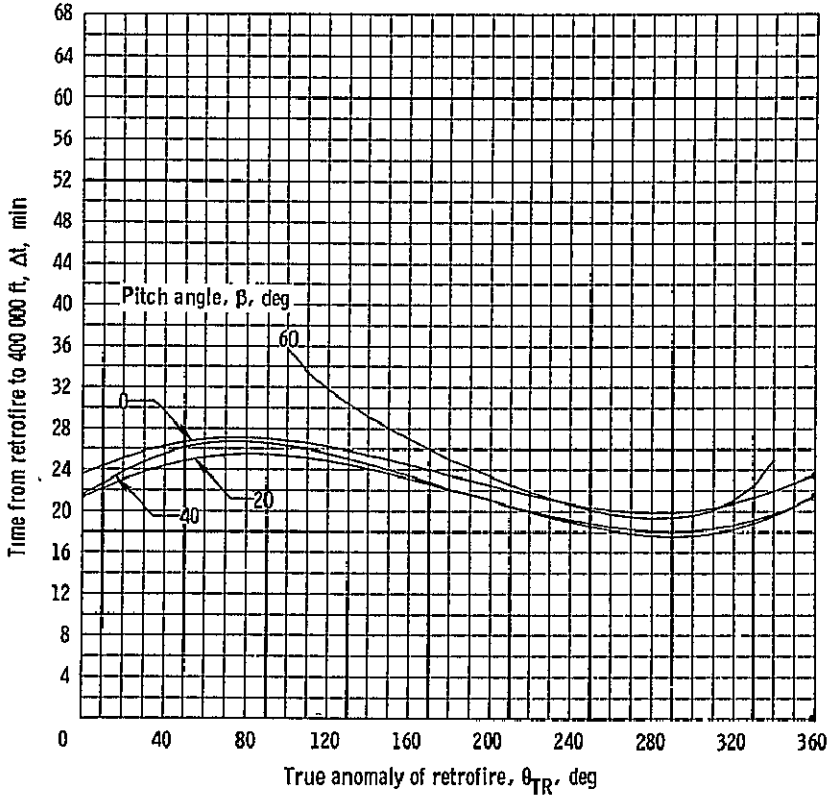
Figure 34.- Concluded.



(a) Retrograde ΔV = 300 fps.

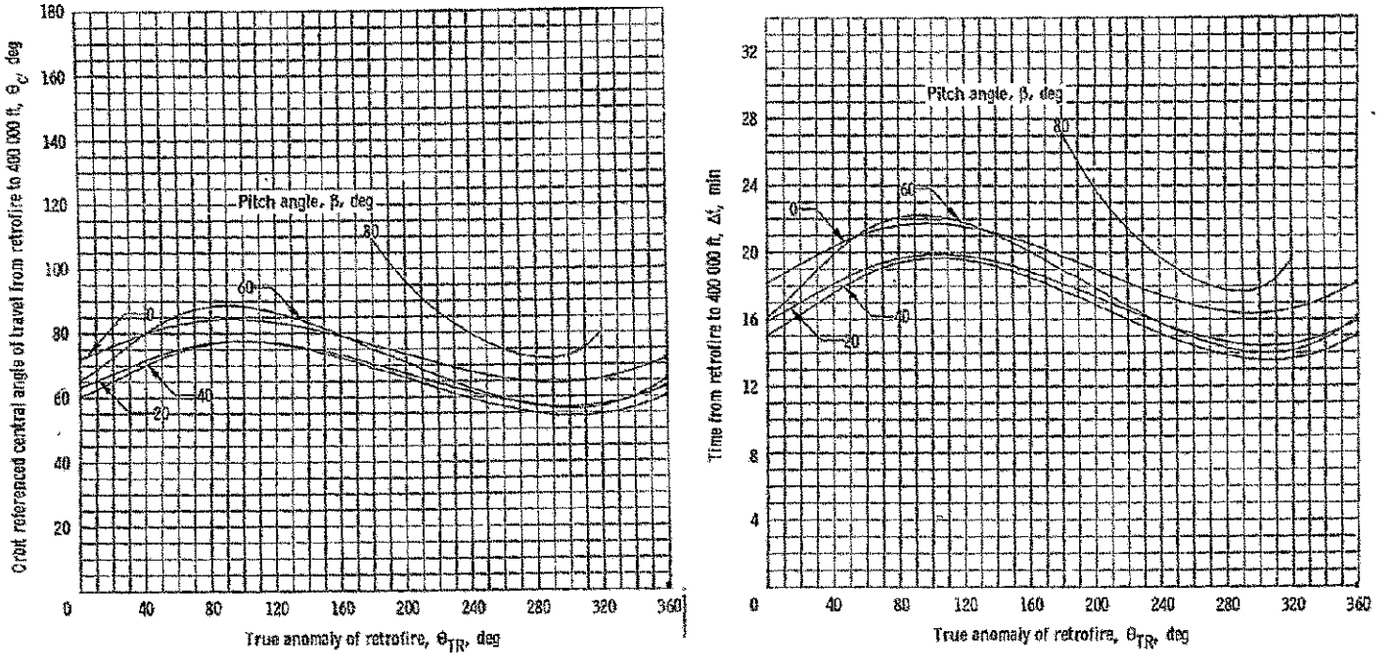
Figure 35. - Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 250 nautical miles and h_p = 180 nautical miles.





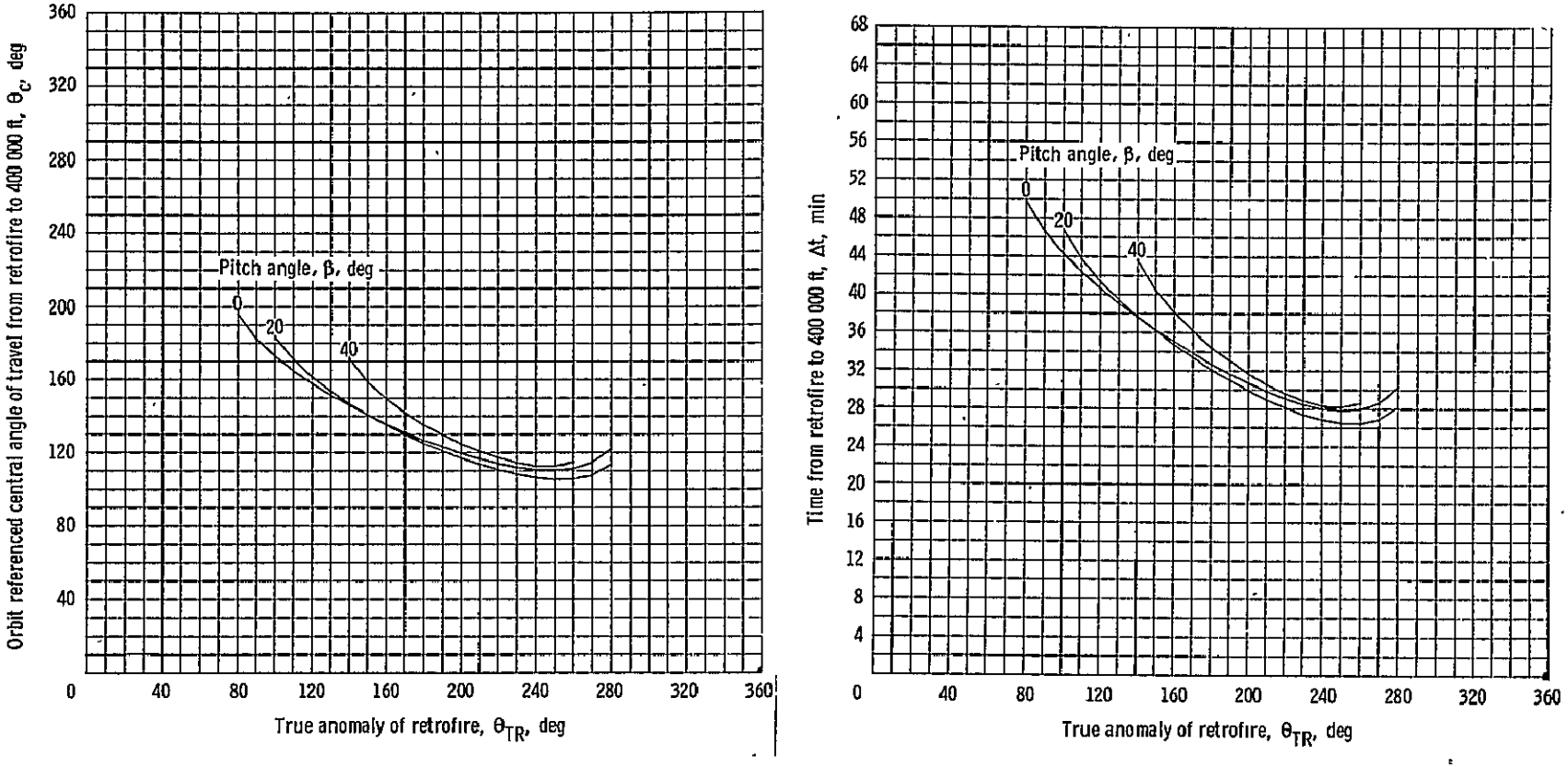
(b) Retrograde ΔV = 500 fps.

Figure 35.– Continued.



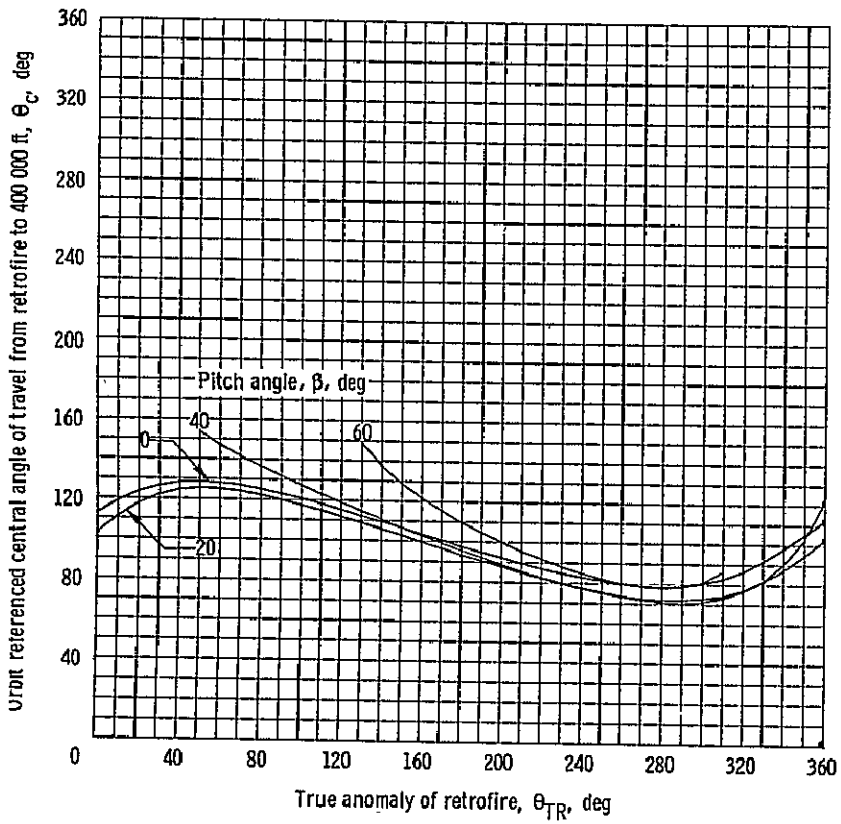
(c) Retrograde ΔV = 700 fps.

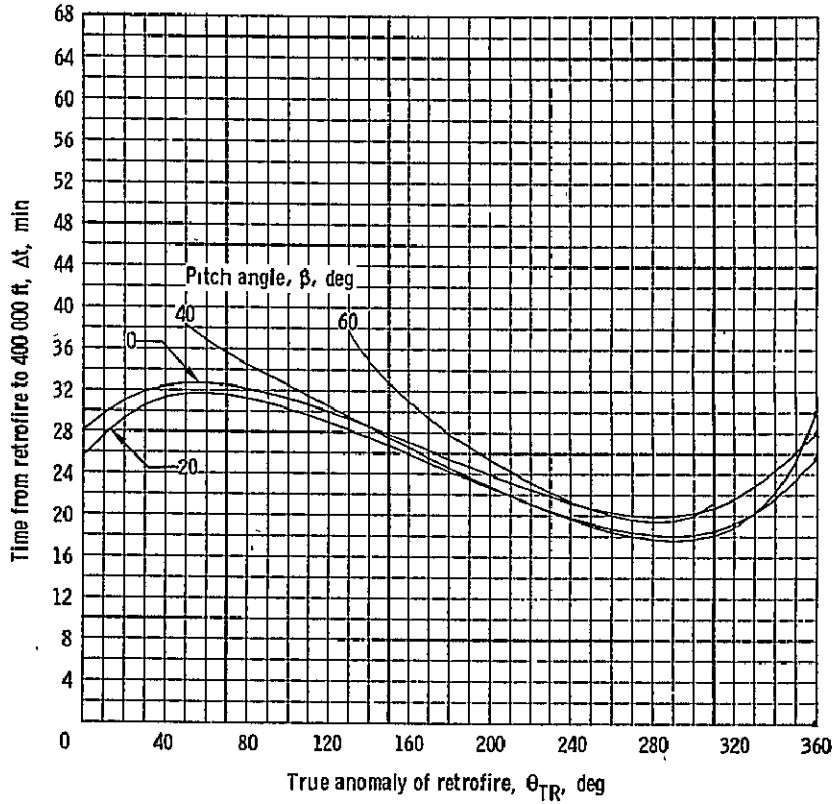
Figure 35. - Concluded.



(a) Retrograde ΔV = 300 fps.

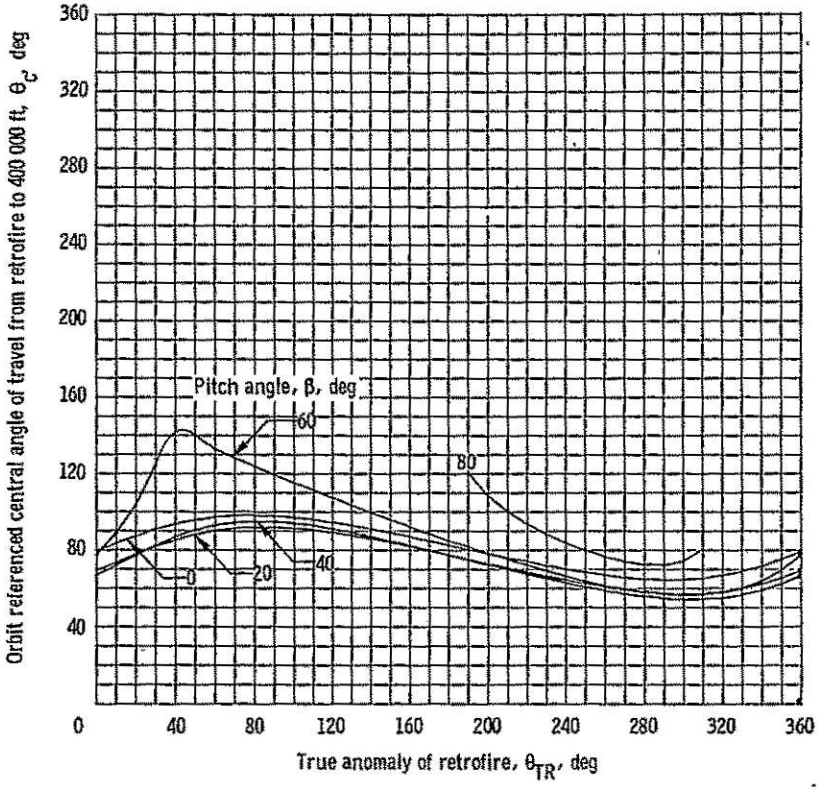
Figure 36. - Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 300 nautical miles and h_p = 180 nautical miles.

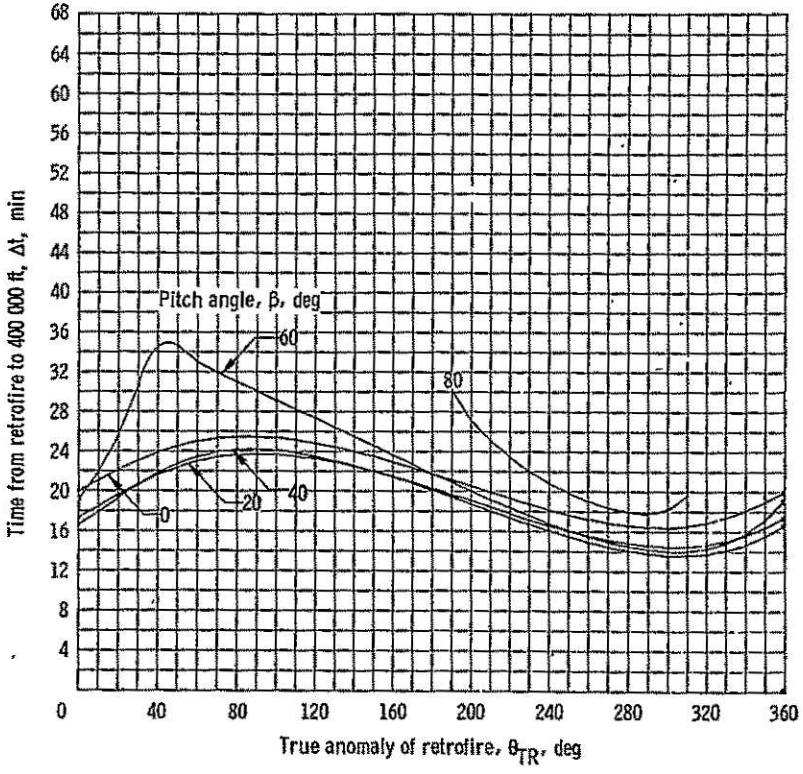




(b) Retrograde ΔV = 500 fps.

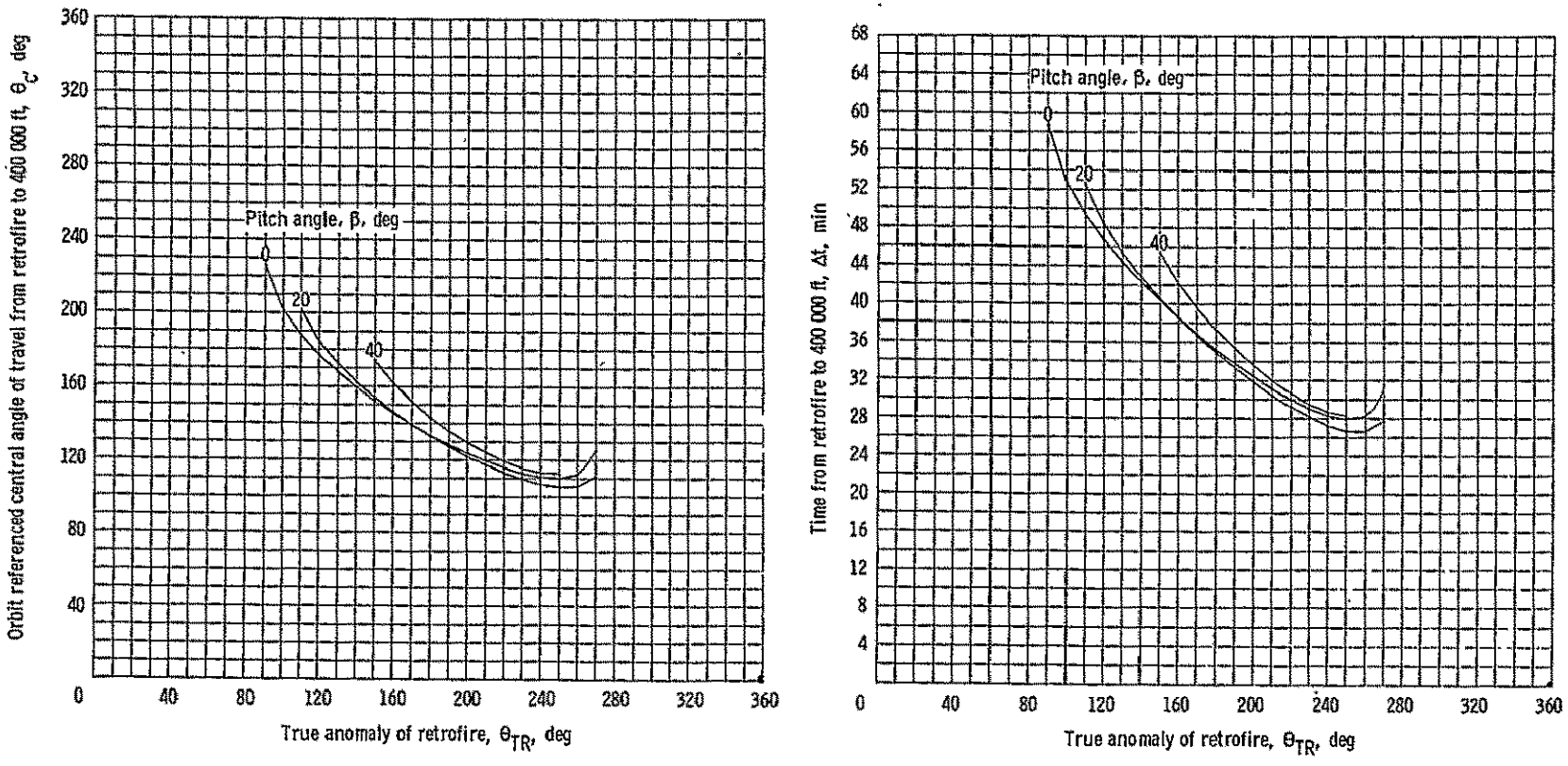
Figure 36. - Continued.





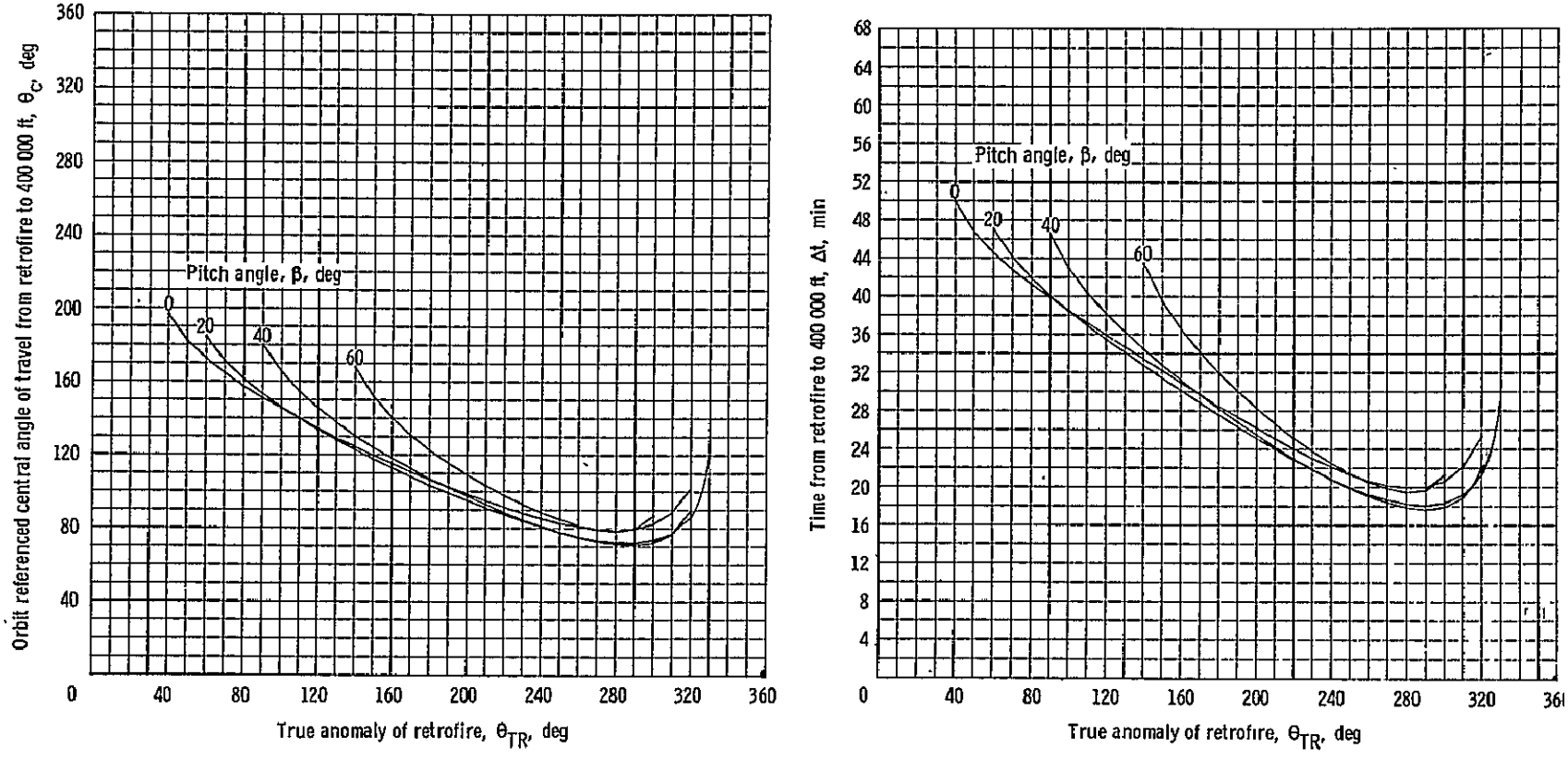
(c) Retrograde ΔV = 700 fps.

Figure 36. - Concluded.



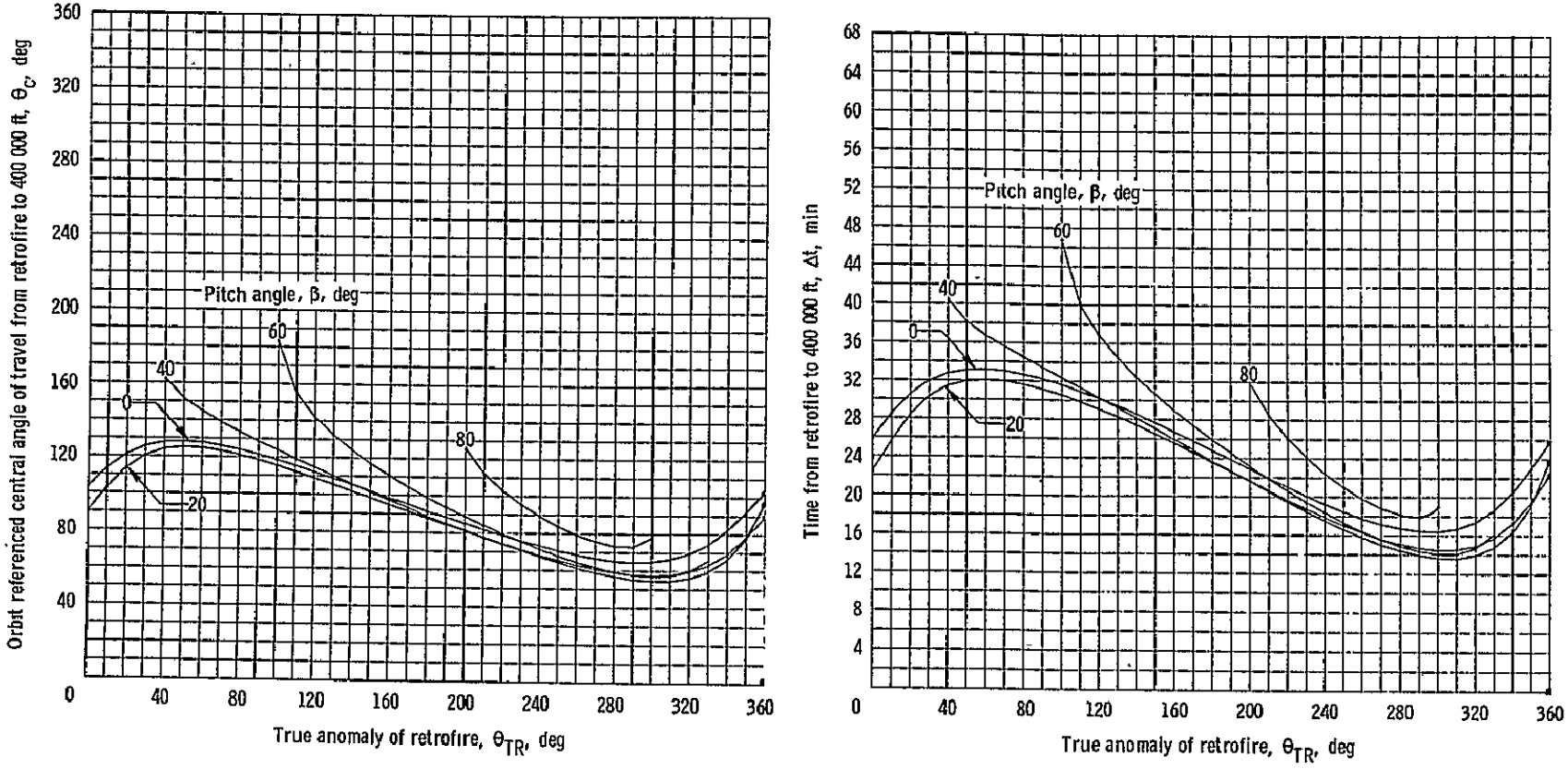
(a) Retrograde ΔV = 300 fps.

Figure 37.- Orbit referenced central angle of travel and time from retrofire to 400 000 feet versus true anomaly of retrofire as functions of various pitch angles for a constant retrograde ΔV; h_a = 400 nautical miles and h_p = 180 nautical miles.



(b) Retrograde ΔV = 500 fps.

Figure 37.- Continued.



(c) Retrograde ΔV = 700 fps.

Figure 37. - Concluded.

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3. Allday, Charles E.: FAB Memorandum 66-FM3-2020, February, 1966.